Matching Theory and Practice

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Overview

1. Introduction
2. Matching Theory
   - Marriage Market
   - Stability
   - Deferred-Acceptance
3. Applications
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   - Kindergarten in Victoria
4. My Research
   - Matching with Quantity
   - Refugee Dispersal
5. Conclusion

Matching Markets

- Money is extremely useful to facilitate transactions
  - Price equilibrates supply and demand
  - Markets organise themselves well
  - Adam Smith's invisible hand
- In matching markets, money cannot be used
  - There may be a price but it does not equilibrate supply and demand
  - These markets do not perform well if left to themselves (market failure)
  - Economists can redesign these markets to make them work better
- Examples
  - School or university admission
  - Kidney donations
  - Allocations of tasks within an organisation
  - Refugee resttlement

A Brief History of Matching

- Gale and Shapley (1962)
  - Brilliant and easy to read paper
  - Theoretical exercise about an abstract marriage market
- Real world applications
  - Started in early 2000's
  - Very active field since then
- 2012 Nobel Prize in Economics
  - Lloyd Shapley and Al Roth
  - "Who Gets What and Why?"

Marriage Market (GS 1962)

- Set of women $\{w_1, w_2, ..., w_n\}$ and set of men $\{m_1, m_2, ..., m_n\}$
  - Each woman can be matched (married) to at most one man
  - Each man can be matched (married) to at most one woman
- People care who they marry
  - Women have (ordinal) preferences over men and remaining single
  - Men have (ordinal) preferences over women and remaining single
- How do we best match these men and women?
  - A key concept is stability
  - It ensures people do not want to rematch
  - Essential to the success of two-sided matching markets

Stability

Definition (Individual Rationality, GS 62)

A matching is **individually rational** if there does not exist any woman or man who would prefer to remain single than to be matched with his/her current partner.

Definition (Stability, GS 62)

A matching is **stable** if it is individually rational there does not exist any woman and any man who would both prefer to be matched with each other than with their current partners.

Example

- Consider the following matching:

$$(w_1, m_1), (w_2, m_2)$$

- **Individual rationality** requires
    - w_1 prefers to be with m_1 than single
    - m_1 prefers to be with w_1 than single
    - w_2 prefers to be with m_2 than single
    - m_2 prefers to be with w_2 than single
- **Stability** requires
    - Individual rationality
    - EITHER w_1 prefers m_1 to m_2 OR m_2 prefers w_2 to w_1
    - EITHER w_2 prefers m_2 to m_1 OR m_1 prefers w_1 to w_2

Deferred-Acceptance Algorithm

- Each man proposes to the woman he prefers (if any)
  - Each woman tentatively accepts her favourite proposal (if any)
  - She rejects all other proposals
- Each man makes a new proposal
  - If he was accepted he proposes to the same woman again
  - If he was rejected he proposes to his next favourite woman (if any)
- The algorithm terminates when all proposals are accepted
  - Each man is matched with the woman to whom he last proposed
  - Each man who did not make a proposal remains single
  - Each woman who did not accept any proposal remains single
- The algorithm is simple and easy to use in practice
  - It can be coded in an Excel spreadsheet (Visual Basics)

Example

Ashley:	$G, E, H, F, \emptyset$	Eric:	$A, \emptyset$
Barbara:	$E, H, \emptyset$	Frank:	$A, C, B, D, \emptyset$
Chelsea:	$F, H, G, \emptyset$	George:	$B, C, D, A, \emptyset$
Dory:	$F, G, H, E, \emptyset$	Henry:	$C, D, A, \emptyset$

Example

Ashley:	$G, E, H, F, \emptyset$	Eric:	$A, \emptyset$
Barbara:	$E, H, \emptyset$	Frank:	$A, C, B, D, \emptyset$
Chelsea:	$F, H, G, \emptyset$	George:	$B, C, D, A, \emptyset$
Dory:	$F, G, H, E, \emptyset$	Henry:	$C, D, A, \emptyset$

Round 1

Eric	$\rightarrow$	Ashley
Frank	$\rightarrow$	Ashley
George	$\rightarrow$	Barbara
Henry	$\rightarrow$	Chelsea

In the first round, each man proposes to his favourite woman.

Example

Ashley:	$G, E, H, F, \emptyset$	Eric:	$A, \emptyset$
Barbara:	$E, H, \emptyset$	Frank:	$A, C, B, D, \emptyset$
Chelsea:	$F, H, G, \emptyset$	George:	$B, C, D, A, \emptyset$
Dory:	$F, G, H, E, \emptyset$	Henry:	$C, D, A, \emptyset$

Round 1

Eric	$\rightarrow$	Ashley	✓
Frank	$\rightarrow$	Ashley	✗
George	$\rightarrow$	Barbara	
Henry	$\rightarrow$	Chelsea	

Ashley chooses Eric over Frank.

Example

Ashley:	$G, E, H, F, \emptyset$	Eric:	$A, \emptyset$
Barbara:	$E, H, \emptyset$	Frank:	$A, C, B, D, \emptyset$
Chelsea:	$F, H, G, \emptyset$	George:	$B, C, D, A, \emptyset$
Dory:	$F, G, H, E, \emptyset$	Henry:	$C, D, A, \emptyset$

Round 1

Eric	$\rightarrow$	Ashley	✓
Frank	$\rightarrow$	Ashley	✗
George	$\rightarrow$	Barbara	✗
Henry	$\rightarrow$	Chelsea	

Barbara rejects Goerge's proposal.

Example

Ashley:	$G, E, H, F, \emptyset$	Eric:	$A, \emptyset$
Barbara:	$E, H, \emptyset$	Frank:	$A, C, B, D, \emptyset$
Chelsea:	$F, H, G, \emptyset$	George:	$B, C, D, A, \emptyset$
Dory:	$F, G, H, E, \emptyset$	Henry:	$C, D, A, \emptyset$

Round 1

Eric	$\rightarrow$	Ashley	✓
Frank	$\rightarrow$	Ashley	✗
George	$\rightarrow$	Barbara	✗
Henry	$\rightarrow$	Chelsea	✓

Chelsea tentatively accepts Harry's proposal.

Example

Ashley:	$G, E, H, F, \emptyset$	Eric:	$A, \emptyset$
Barbara:	$E, H, \emptyset$	Frank:	$A, C, B, D, \emptyset$
Chelsea:	$F, H, G, \emptyset$	George:	$B, C, D, A, \emptyset$
Dory:	$F, G, H, E, \emptyset$	Henry:	$C, D, A, \emptyset$

Round 1

Eric	$\rightarrow$	Ashley	✓
Frank	$\rightarrow$	Ashley	✗
George	$\rightarrow$	Barbara	✗
Henry	$\rightarrow$	Chelsea	✓

Example

Ashley:	$G, E, H, F, \emptyset$	Eric:	$A, \emptyset$
Barbara:	$E, H, \emptyset$	Frank:	$A, C, B, D, \emptyset$
Chelsea:	$F, H, G, \emptyset$	George:	$B, C, D, A, \emptyset$
Dory:	$F, G, H, E, \emptyset$	Henry:	$C, D, A, \emptyset$

Round 1

Eric	$\rightarrow$	Ashley	✓
Frank	$\rightarrow$	Ashley	✗
George	$\rightarrow$	Barbara	✗
Henry	$\rightarrow$	Chelsea	✓

In Round 2, Frank and George will propose to Chelsea.

Example

Ashley:	$G, E, H, F, \emptyset$	Eric:	$A, \emptyset$
Barbara:	$E, H, \emptyset$	Frank:	$A, C, B, D, \emptyset$
Chelsea:	$F, H, G, \emptyset$	George:	$B, C, D, A, \emptyset$
Dory:	$F, G, H, E, \emptyset$	Henry:	$C, D, A, \emptyset$

Round 2

Eric	$\rightarrow$	Ashley
Frank	$\rightarrow$	Chelsea
George	$\rightarrow$	Chelsea
Henry	$\rightarrow$	Chelsea

Example

Ashley:	$G, E, H, F, \emptyset$	Eric:	$A, \emptyset$
Barbara:	$E, H, \emptyset$	Frank:	$A, C, B, D, \emptyset$
Chelsea:	$F, H, G, \emptyset$	George:	$B, C, D, A, \emptyset$
Dory:	$F, G, H, E, \emptyset$	Henry:	$C, D, A, \emptyset$

Round 2

Eric	$\rightarrow$	Ashley	✓
Frank	$\rightarrow$	Chelsea	
George	$\rightarrow$	Chelsea	
Henry	$\rightarrow$	Chelsea	

Ashley again tentatively accepts Eric's proposal.

Example

Ashley:	$G, E, H, F, \emptyset$	Eric:	$A, \emptyset$
Barbara:	$E, H, \emptyset$	Frank:	$A, C, B, D, \emptyset$
Chelsea:	$F, H, G, \emptyset$	George:	$B, C, D, A, \emptyset$
Dory:	$F, G, H, E, \emptyset$	Henry:	$C, D, A, \emptyset$

Round 2

Eric	$\rightarrow$	Ashley	✓
Frank	$\rightarrow$	Chelsea	✓
George	$\rightarrow$	Chelsea	✗
Henry	$\rightarrow$	Chelsea	✗

Chelsea chooses Frank over George and Henry.

Example

Ashley:	$G, E, H, F, \emptyset$	Eric:	$A, \emptyset$
Barbara:	$E, H, \emptyset$	Frank:	$A, C, B, D, \emptyset$
Chelsea:	$F, H, G, \emptyset$	George:	$B, C, D, A, \emptyset$
Dory:	$F, G, H, E, \emptyset$	Henry:	$C, D, A, \emptyset$

Round 2

Eric	$\rightarrow$	Ashley	✓
Frank	$\rightarrow$	Chelsea	✓
George	$\rightarrow$	Chelsea	✗
Henry	$\rightarrow$	Chelsea	✗

Example

Ashley:	$G, E, H, F, \emptyset$	Eric:	$A, \emptyset$
Barbara:	$E, H, \emptyset$	Frank:	$A, C, B, D, \emptyset$
Chelsea:	$F, H, G, \emptyset$	George:	$B, C, D, A, \emptyset$
Dory:	$F, G, H, E, \emptyset$	Henry:	$C, D, A, \emptyset$

Round 2

Eric	$\rightarrow$	Ashley	✓
Frank	$\rightarrow$	Chelsea	✓
George	$\rightarrow$	Chelsea	✗
Henry	$\rightarrow$	Chelsea	✗

In Round 3, George and Henry will propose to Dory.

Example

Ashley:	$G, E, H, F, \emptyset$	Eric:	$A, \emptyset$
Barbara:	$E, H, \emptyset$	Frank:	$A, C, B, D, \emptyset$
Chelsea:	$F, H, G, \emptyset$	George:	$B, C, D, A, \emptyset$
Dory:	$F, G, H, E, \emptyset$	Henry:	$C, D, A, \emptyset$

Round 3

Eric	$\rightarrow$	Ashley
Frank	$\rightarrow$	Chelsea
George	$\rightarrow$	Dory
Henry	$\rightarrow$	Dory

Example

Ashley:	$G, E, H, F, \emptyset$	Eric:	$A, \emptyset$
Barbara:	$E, H, \emptyset$	Frank:	$A, C, B, D, \emptyset$
Chelsea:	$F, H, G, \emptyset$	George:	$B, C, D, A, \emptyset$
Dory:	$F, G, H, E, \emptyset$	Henry:	$C, D, A, \emptyset$

Round 3

Eric	$\rightarrow$	Ashley	✓
Frank	$\rightarrow$	Chelsea	
George	$\rightarrow$	Dory	
Henry	$\rightarrow$	Dory	

Ashley tentatively accept Eric's proposal.

Example

Ashley:	$G, E, H, F, \emptyset$	Eric:	$A, \emptyset$
Barbara:	$E, H, \emptyset$	Frank:	$A, C, B, D, \emptyset$
Chelsea:	$F, H, G, \emptyset$	George:	$B, C, D, A, \emptyset$
Dory:	$F, G, H, E, \emptyset$	Henry:	$C, D, A, \emptyset$

Round 3

Eric	$\rightarrow$	Ashley	✓
Frank	$\rightarrow$	Chelsea	✓
George	$\rightarrow$	Dory	
Henry	$\rightarrow$	Dory	

Chelsea tentatively accept Frank's proposal.

Example

Ashley:	$G, E, H, F, \emptyset$	Eric:	$A, \emptyset$
Barbara:	$E, H, \emptyset$	Frank:	$A, C, B, D, \emptyset$
Chelsea:	$F, H, G, \emptyset$	George:	$B, C, D, A, \emptyset$
Dory:	$F, G, H, E, \emptyset$	Henry:	$C, D, A, \emptyset$

Round 3

Eric	$\rightarrow$	Ashley	✓
Frank	$\rightarrow$	Chelsea	✓
George	$\rightarrow$	Dory	✓
Henry	$\rightarrow$	Dory	✗

Dory chooses George over Henry.

Example

Ashley:	$G, E, H, F, \emptyset$	Eric:	$A, \emptyset$
Barbara:	$E, H, \emptyset$	Frank:	$A, C, B, D, \emptyset$
Chelsea:	$F, H, G, \emptyset$	George:	$B, C, D, A, \emptyset$
Dory:	$F, G, H, E, \emptyset$	Henry:	$C, D, A, \emptyset$

Round 3

Eric	$\rightarrow$	Ashley	✓
Frank	$\rightarrow$	Chelsea	✓
George	$\rightarrow$	Dory	✓
Henry	$\rightarrow$	Dory	✗

Example

Ashley:	$G, E, H, F, \emptyset$	Eric:	$A, \emptyset$
Barbara:	$E, H, \emptyset$	Frank:	$A, C, B, D, \emptyset$
Chelsea:	$F, H, G, \emptyset$	George:	$B, C, D, A, \emptyset$
Dory:	$F, G, H, E, \emptyset$	Henry:	$C, D, A, \emptyset$

Round 3

Eric	$\rightarrow$	Ashley	✓
Frank	$\rightarrow$	Chelsea	✓
George	$\rightarrow$	Dory	✓
Henry	$\rightarrow$	Dory	✗

In Round 4, Henry will propose to Ashley.

Example

Ashley:	$G, E, H, F, \emptyset$	Eric:	$A, \emptyset$
Barbara:	$E, H, \emptyset$	Frank:	$A, C, B, D, \emptyset$
Chelsea:	$F, H, G, \emptyset$	George:	$B, C, D, A, \emptyset$
Dory:	$F, G, H, E, \emptyset$	Henry:	$C, D, A, \emptyset$

Round 4

Eric	$\rightarrow$	Ashley
Frank	$\rightarrow$	Chelsea
George	$\rightarrow$	Dory
Henry	$\rightarrow$	Ashley

Example

Ashley:	$G, E, H, F, \emptyset$	Eric:	$A, \emptyset$
Barbara:	$E, H, \emptyset$	Frank:	$A, C, B, D, \emptyset$
Chelsea:	$F, H, G, \emptyset$	George:	$B, C, D, A, \emptyset$
Dory:	$F, G, H, E, \emptyset$	Henry:	$C, D, A, \emptyset$

Round 4

Eric	$\rightarrow$	Ashley	✓
Frank	$\rightarrow$	Chelsea	
George	$\rightarrow$	Dory	
Henry	$\rightarrow$	Ashley	✗

Ashley chooses Eric over Henry.

Example

Ashley:	$G, E, H, F, \emptyset$	Eric:	$A, \emptyset$
Barbara:	$E, H, \emptyset$	Frank:	$A, C, B, D, \emptyset$
Chelsea:	$F, H, G, \emptyset$	George:	$B, C, D, A, \emptyset$
Dory:	$F, G, H, E, \emptyset$	Henry:	$C, D, A, \emptyset$

Round 4

Eric	→	Ashley	✓
Frank	→	Chelsea	✓
George	→	Dory	✓
Henry	→	Ashley	✗

Chelsea and Dory tentatively accept their respective proposals.

Example

Ashley:	$G, E, H, F, \emptyset$	Eric:	$A, \emptyset$
Barbara:	$E, H, \emptyset$	Frank:	$A, C, B, D, \emptyset$
Chelsea:	$F, H, G, \emptyset$	George:	$B, C, D, A, \emptyset$
Dory:	$F, G, H, E, \emptyset$	Henry:	$C, D, A, \emptyset$

Round 4

Eric	$\rightarrow$	Ashley	✓
Frank	$\rightarrow$	Chelsea	✓
George	$\rightarrow$	Dory	✓
Henry	$\rightarrow$	Ashley	✗

Example

Ashley:	$G, E, H, F, \emptyset$	Eric:	$A, \emptyset$
Barbara:	$E, H, \emptyset$	Frank:	$A, C, B, D, \emptyset$
Chelsea:	$F, H, G, \emptyset$	George:	$B, C, D, A, \emptyset$
Dory:	$F, G, H, E, \emptyset$	Henry:	$C, D, A, \emptyset$

Round 4

Eric	$\rightarrow$	Ashley	✓
Frank	$\rightarrow$	Chelsea	✓
George	$\rightarrow$	Dory	✓
Henry	$\rightarrow$	Ashley	✗

Henry has run out of options and will not make any proposal in Round 5.

Example

Ashley:	$G, E, H, F, \emptyset$	Eric:	$A, \emptyset$
Barbara:	$E, H, \emptyset$	Frank:	$A, C, B, D, \emptyset$
Chelsea:	$F, H, G, \emptyset$	George:	$B, C, D, A, \emptyset$
Dory:	$F, G, H, E, \emptyset$	Henry:	$C, D, A, \emptyset$

Round 5

Eric	$\rightarrow$	Ashley
Frank	$\rightarrow$	Chelsea
George	$\rightarrow$	Dory
Henry	$\rightarrow$	$\emptyset$

Example

Ashley:	$G, E, H, F, \emptyset$	Eric:	$A, \emptyset$
Barbara:	$E, H, \emptyset$	Frank:	$A, C, B, D, \emptyset$
Chelsea:	$F, H, G, \emptyset$	George:	$B, C, D, A, \emptyset$
Dory:	$F, G, H, E, \emptyset$	Henry:	$C, D, A, \emptyset$

Round 5

Eric	$\rightarrow$	Ashley	✓
Frank	$\rightarrow$	Chelsea	✓
George	$\rightarrow$	Dory	✓
Henry	$\rightarrow$	$\emptyset$	✓

All proposals are accepted and the algorithm terminates.

Example

Ashley:	$G, E, H, F, \emptyset$	Eric:	$A, \emptyset$
Barbara:	$E, H, \emptyset$	Frank:	$A, C, B, D, \emptyset$
Chelsea:	$F, H, G, \emptyset$	George:	$B, C, D, A, \emptyset$
Dory:	$F, G, H, E, \emptyset$	Henry:	$C, D, A, \emptyset$

Outcome

Eric is matched with Ashley
Frank is matched with Chelsea
George is matched with Dory
Henry and Barbara remain unmatched

Example

Ashley:	$G, E, H, F, \emptyset$	Eric:	$A, \emptyset$
Barbara:	$E, H, \emptyset$	Frank:	$A, C, B, D, \emptyset$
Chelsea:	$F, H, G, \emptyset$	George:	$B, C, D, A, \emptyset$
Dory:	$F, G, H, E, \emptyset$	Henry:	$C, D, A, \emptyset$

Summary

Round 1	Round 2	Round 3	Round 4	Round 5
$E \rightarrow A$ ✓	$E \rightarrow A$ ✓	$E \rightarrow A$ ✓	$E \rightarrow A$ ✓	$E \rightarrow A$ ✓
$F \rightarrow A$ ✗	$F \rightarrow C$ ✓	$F \rightarrow C$ ✓	$F \rightarrow C$ ✓	$F \rightarrow C$ ✓
$G \rightarrow B$ ✗	$G \rightarrow C$ ✗	$G \rightarrow D$ ✓	$G \rightarrow D$ ✓	$G \rightarrow D$ ✓
$H \rightarrow C$ ✓	$H \rightarrow C$ ✗	$H \rightarrow D$ ✗	$H \rightarrow A$ ✗	$H \rightarrow \emptyset$ ✓

Properties of DA

Theorem

The matching produced by the man-proposing deferred-acceptance algorithm is the man-optimal stable matching.

- Man-optimal stable matching
  - In any other stable matching, all men are either matched with the same woman or with one they like less
  - Best stable matching from the men's point of view
  - Worst stable matching from the women's point of view

Properties of DA

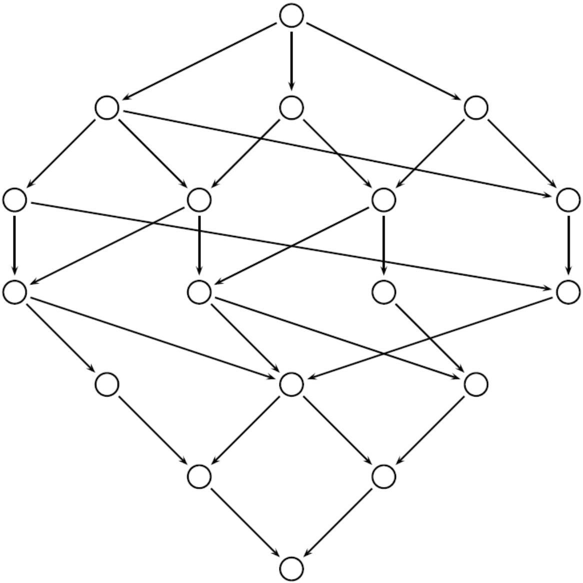
Theorem

The matching produced by the woman-proposing deferred-acceptance algorithm is the woman-optimal stable matching.

- Woman-optimal stable matching
  - In any other stable matching, all women are either matched with the same man or with one they like less
  - Best stable matching from the women's point of view
  - Worst stable matching from the men's point of view

Set of Stable Matchings

- On one extreme, men-optimal stable matching.
  - Found by the men-proposing DA
  - Best stable matching for men, worst for women
- On the other extreme, woman-optimal stable matching
  - Found by the women-proposing DA
  - Best stable matching for women, worst for men
- The set of stable matching is always nonempty
  - If both versions of DA give the same matching: unique stable matching
  - Otherwise DA gives the two extremes
  - There may be more stable matchings in between



Incentive Properties

Theorem (GS 62)

The deferred acceptance is **strategy-proof** *for the* **proposing** *side but not for the proposed side.*

- In the men-proposing deferred-acceptance algorithm:
  - Men can only lose out if they misrepresent their preferences
  - Women can potentially gain by misrepresenting their preferences
- Finding the right strategy is difficult and risky
  - More likely to lose than gain
  - Generally not regarded as a big problem

Overview

From Theory To Practice

- Gale and Shapley (1962) proposed a theoretical model
  - To the best of my knowledge no marriage is arranged in this way
  - Mathematics and romance do not always get along...
  - The literature remained essentially theoretical until the early 2000's
- Since then many applications
  - **School Choice**
  - Kidney Exchange
  - National Resident Matching Program
- This presentation focuses on school choice
  - It constitutes the starting point of the applied matching literature
  - The problem is similar to the marriage market
  - It is relevant to Victoria

School Choice

- Abdulkadiroglu and Sonmez (2003)
  - Excellent paper, easy to read
  - High school students assigned to schools in Boston
  - It has been extended to many US cities
  - It could be applied to Melbourne
- Assigning students to their neighbourhood causes problems
  - Wealthy parents move to areas with good schools
  - United States cities are very segregated
- Allowing students to choose has three advantages
  - It is welfare enhancing
  - It reduces the importance of family wealth
  - It mixes populations

The Problem

- School Choice existed but was not done optimally
  - Ineffective “Boston” algorithm
  - Incentive problem and unfair matching
- The authors proposed a new design
  - Based on Gale and Shapley (1962)
  - Uses the deferred acceptance algorithm
- The new design was implemented
  - Economists have been designing matching markets ever since

The Model

- Very close to the marriage market
  - Set of students and set of schools
  - Students have ordinal preferences over schools
  - Schools have ordinal *priorities* over students
- Many-to-one matching
  - Each school is matched with many students
  - Each school has a capacity limit (number of students it can fit)
  - This hardly makes a difference, GS 62 considered it as an extension
- The market is one-sided
  - Schools are not strategic agents, school seats are goods
  - Only students' welfare matters
  - Schools priorities (not preferences) are determined by law
  - This is important

One-Sided Market

- Priorities determined by law
  - Higher priority if the school is in the same neighbourhood
  - Higher priority if the sibbling is attending the school
  - Lottery
- Stability means fairness
  - Schools are not strategic agent, they will not rematch
  - A stable matching is fair: if a student misses out on a school (s)he likes, then all students attending that school have a higher priority
- Student proposing DA has desirable properties
  - It is strategy-proof
  - It is stable (fair)
  - It maximises welfare given the stability (fairness) constraint

Policy Implications

- In the United States
  - "Boston" algorithm was replaced by deferred acceptance
  - Similar designs were implemented in other cities
- Can we learn from this in Victoria?
  - Kindergarten
  - Schools?
  - Child Care?

Kindergarten in Victoria

- What is kindergarten?
  - Often called *Preschool*
  - One year program, two years before Grade 1
  - Attendance is optional and places are not guaranteed
  - Funded by the state, often owned and operated by councils
  - Sometimes privately owned but strictly regulated
- A matching market
  - Children (or their parents) have preferences over kindergarten
  - Priorities for each kindergarten are determined by law
  - Each kindergarten has a capacity limit
  - The problem is almost identical to school choice

Using Matching Theory

- Typical process
  - Centralised at the council level
  - Four rounds of offers over two months
  - Outcome is similar to the “Boston” algorithm...
  - But it takes two months instead of thirty seconds
- It could be replaced by the deferred acceptance algorithm
  - Large amount of time and paperwork saved
  - Better allocation
  - Strategy-proof for families
  - Better information on demand for kindergarten

Obstacles

- People like to be in control
  - They are rightfully weary of mysterious algorithms
  - Explaining how it works goes a long way
- Councils may feel power is taken away from them
  - They retain control over priorities
  - They continue to manage kindergartens
  - Only the headaches associated with the matching are taken away
- People do not like change
  - Start with a pilot in one or two councils

Overview

Childcare Matching

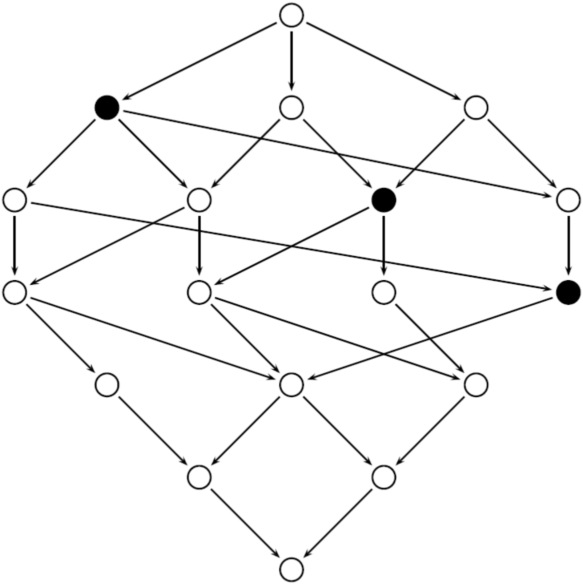
- A matching market
  - Families have preferences over childcare centres
  - Priorities are determined by law (centres can to some extent have a say)
  - Each centre has a capacity limit
- Two main differences with kindergarten
  - Children can enter or leave at any point
  - Children can attend part-time
- Dynamic issue
  - Trade-off in terms of how often the market is cleared
  - Thicker market vs waiting time
- Part-time issue
  - Enormous consequences on the model
  - This is what I study in my paper

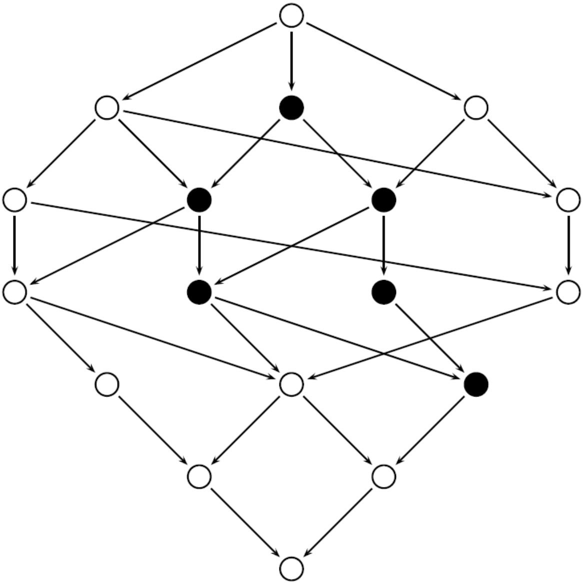
Matching with Quantity

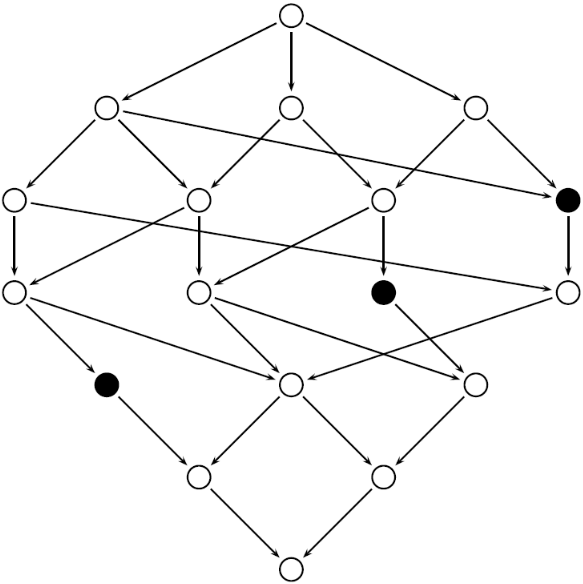
- Simple model
  - Focuses on the heart of the problem
  - Any application, including childcare is inevitably more complex
  - The main insights developed are still valid
- Some agents want two units of the same good
  - These agents are not interested in getting just one unit
  - Children who need to attend childcare full-time
- Some agents only want one unit
  - Children who need to attend part-time
- Complementarity in preferences
  - An agent who wants two units sees them as complements
  - A unit is worth more to the agent if (s)he already has one
  - This is the heart of the problem

Consequences of Complementarity

- The set of stable matching is not well behaved
  - It may be empty
  - It may not contain an agent-optimal stable matching
  - Instead there may be several undominated ones
- The deferred acceptance algorithm does not work
  - Even if an agent-optimal stable matching exists it may not find it
- What is going on?
  - The set of stable matching is part of a larger set
  - That set is well behaved
- Relax the definition of stability
  - Allow for some degree infeasibility and instability
  - "Pseudo-stable" matchings
  - Seach that well-behaved set for stable matchings







Finding Stable Matchings

- The algorithm works in two stages
  - (i) Adapt the deferred acceptance algorithm to find the agent-optimal pseudo-stable matching
  - (ii) Search the set to find stable matchings
- All stable matchings can be found in this way
  - This can be computationally heavy
  - Finding an undominated stable matching may be enough
- Applications
  - Childcare matching
  - University exchange programs
  - Matching with couples
  - **Refugee dispersal**

Refugee Dispersal

- Joint project
  - Scott Kominers (Harvard)
  - Alex Teytelboym (Oxford)
- The United Kingdom will resettle 20,000 Syrian refugees by 2020
  - These will be spread across the country in several localities
- We study this matching market
  - Refugees have preferences over localities
  - Localities can set up priorities
  - Localities have capacity limits

Refugee Dispersal

- Refugees are more likely to sucessfully integrate if
    - They are relocated in a place they like
    - They have the services they need
    - They have a chance to find work
    - They are a good fit for the community
- Technical Difficulty
    - Families have different sizes
    - Families require different services (schools, hospitals, etc)
- Complex version of the quantity problem
    - A similar algorithm can be found to find a stable matching

International Refugee Cirisis

- Refugees currently have three options
  - Apply to one country at a time
  - Wait around in a camp to be processed by the UN
  - Reach Europe (or Australia) and claim asylum
- This could be organised as a matching market
  - Refugees have preferences over countries
  - Countries have preferences over refugees and set quotas
  - Quantity problem does not matter on such a large scale
- This is a standard two-sided matching market
  - Deferred acceptance works well
  - The hard part is to convince countries to offer resettlement places
  - The quotas must be high enough for refugees to enter the system rather than seek asylum

Conclusion

- Matching theory has many applications
  - School choice, kidney exchange, labour market, university admission, doctor-hospital matching, cadet matching, refugee dispersal, etc
  - Organising these markets efficiently can make a real difference
- Research continues
  - More complex matching models and algorithms are being developed
  - Potential for more applications
- The current theory already has great potential
  - It is underutilised and many markets could be improved
- Academics have little incentive to tackle these problems
  - This is the purpose of the Center for Market Design
  - Public servants have a very important role to play

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