CENTRE FOR MARKET DESIGN

A CONSORTIUM OF

THE UNIVERSITY OF MELBOURNE | FACULTY OF BUSINESS & ECONOMICS

Australian Government The Treasury

Victoria Department of Treasury and Finance

LECTURE 12: MATCHING APPLICATIONS

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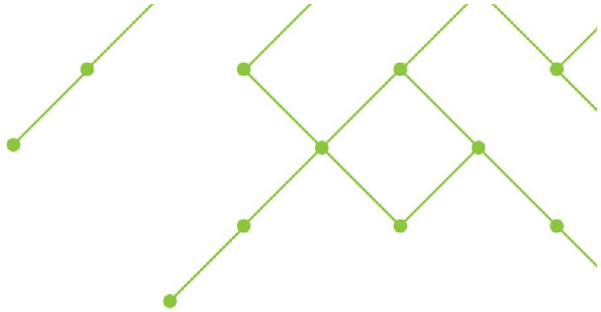
4 DECEMBER 2014

Introduction



- Matching theory can be applied to a wide range of situations.
  - School Choice.
  - Resident Matching Program.
  - Kidney Exchange.

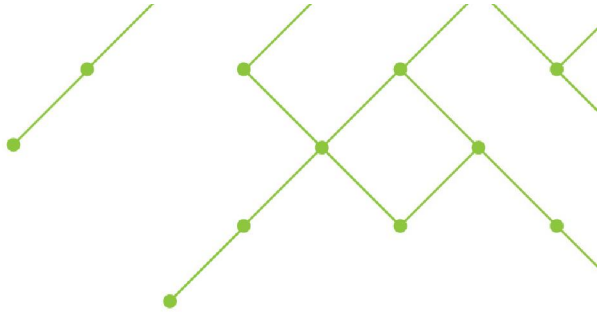
- This lecture is devoted to three applications in Victoria.
  - Allocation of kindergarten places.
  - DTF job transfer program.
  - Tertiary education admission.

Kindergarten in Victoria

What is Kindergarten?

- Kindergarten is a one-year early educational program.
  - Often called *Preschool*.
  - Children must be 4 by 30 April of the year they attend.
  - It takes place two years before Grade 1.
  - The program includes a minimum of 15 hours per week.
  - Attending kindergarten is optional and children are not guaranteed a place.
  - Kindergartens are funded by the State and often owned and operated by local councils.
  - They may be privately owned and operated but must follow strict regulation in order to get funding.

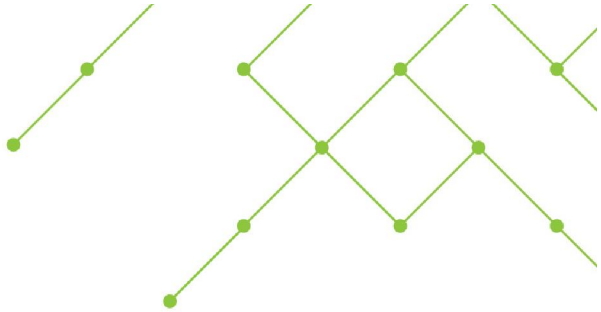
A Matching Model



- Kindergarten provides an interesting matching model
  - Similar to the well-known school choice model.
  - A child goes to at most one kindergarten and is either enrolled or not enrolled (no part-time).
  - Each kindergarten can accommodate a limited number of children.
  - Parents have preferences over kindergartens (location).
  - Children have different priorities at each kindergarten.
  - The market is *one-sided* as kindergartens are not agents.

- Main difference with school choice.
  - Children do not need to be matched.

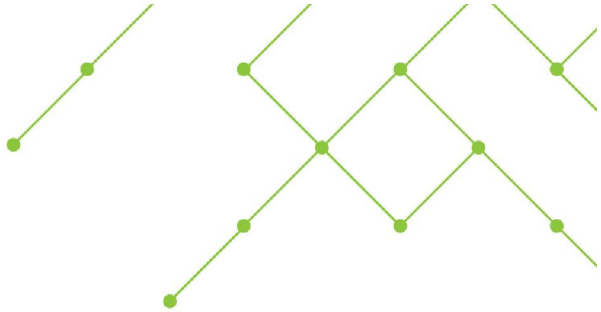
State Priorities



- Following Commonwealth guidelines, the State of Victoria recommends the following groups be given high priority:
  - Children at risk of neglect or abuse.
  - Aboriginal and/or Torres Straight Islander children.
  - Children with additional needs (e.g. disability).

- Another recommendation is to avoid any discriminatory rule.
  - For example, sex, race or age should not be taken into account.
  - Time of application cannot be used either if parents can only apply once the child has reached a certain age.

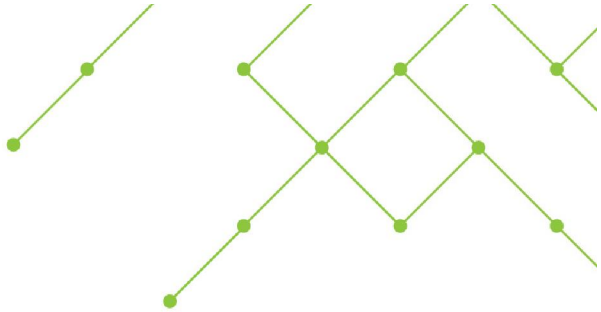
Local Priorities



- Each kindergarten manager can choose their own rules.
  - They should favour the high priority groups.
  - They should not be discriminatory.
  - They must be communicated to families (no arbitrary priority).

- Priority criteria may include.
  - Having a sibling at school or childcare in the same building.
  - Living close to the kindergarten or within the same council.
  - A lottery to break children who are tied.

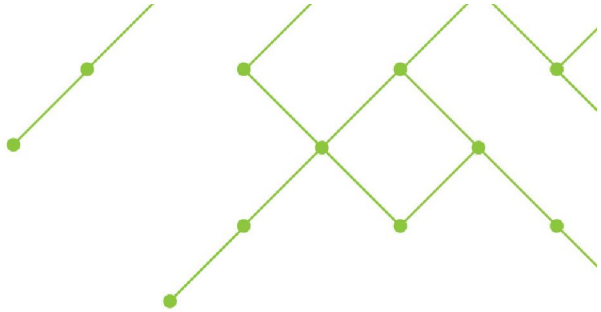
An Example: Darebin



- Three categories:
  - Children with additional needs and vulnerable families.
  - Children who currently use the same service.
  - All other children.

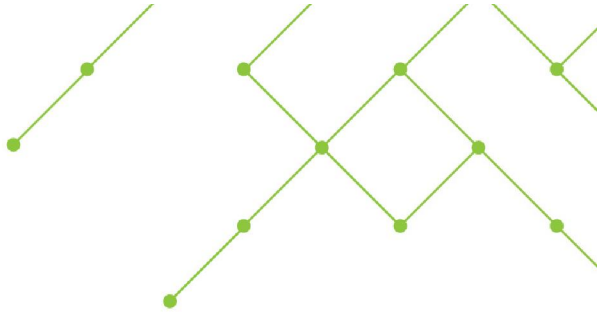
- Within each category, priorities depend on a point system.
  - 50 points if the family lives in Darebin or the child is attending childcare in Darebin.
  - 30 points if the child has a sibling who attended the preferred kindergarten within the last two years.
  - 20 points if the kindergarten is the closest to the child's home.
  - Computer generated random numbers break ties.

An Example: Bendigo



- Four categories:
  - Children who qualify for a second year of kindergarten or who have developmental delays or disabilities.
  - Parent's preference of kindergarten as on the application form.
  - Children who attended pre-kindergarten the year before at the same location.
  - Children with siblings who attended the service within the last three years.
- Within a category: eldest to youngest.
  - This goes against the State's recommendations.

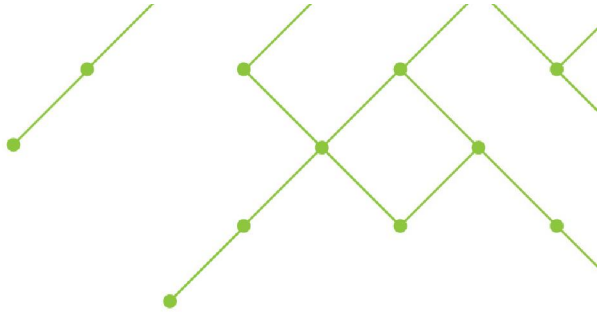
Decentralised Matching



- Ten years ago, the application system was decentralised.
  - Parents had to apply separately to each kindergarten.
  - Each kindergarten managed its own enrolments.

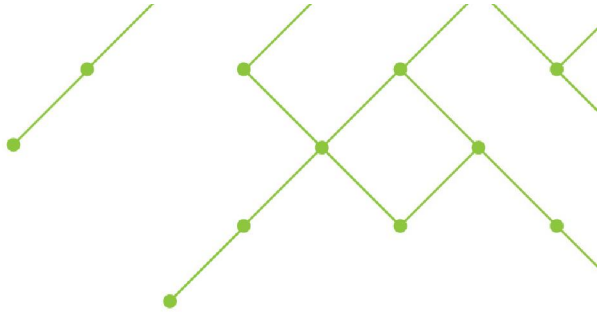
- **Coordination problem**.
  - If a family gets multiple offers, they can accept at most one and reject all others.
  - New slots are liberated, new offers are made.
  - Introduction of waiting lists.

DM is not optimal



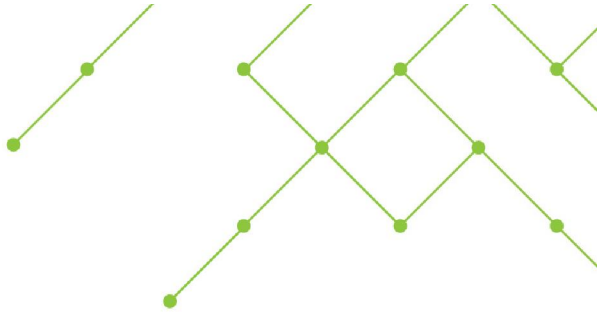
- Hard choice for parents
  - Accept an offer or wait for a better one?
- Fairness Concern
  - Priorities are not necessarily respected.
- Inefficiencies
  - Families may not get their best possible outcome.
  - Large amount of paperwork for families and kindergartens.

Centralisation



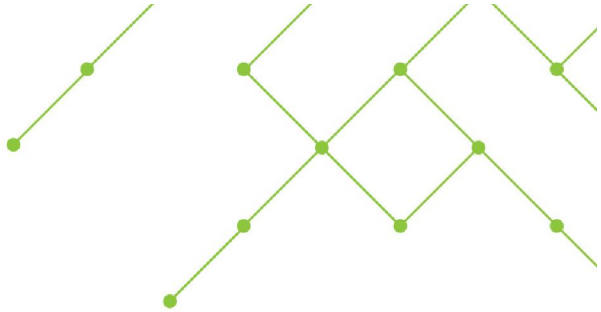
- Economists dislike that word.
  - It feels at odds with free market economics.
- Centralisation can however be a sensible economic policy.
  - Market Failure.
- In the kindergarten sector, there is no price equilibrium.
  - Fees are regulated so that everyone can afford them.
  - Priorities are based on equity concerns.
  - No invisible hand to equate demand and supply.

Centralisation and Timing



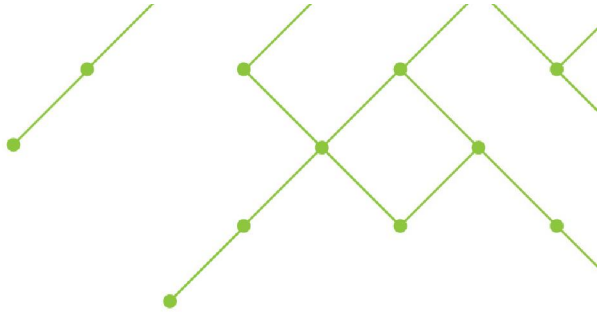
- Centralisation is more difficult in a dynamic market.
  - If the market is cleared too often, the number of people involved might be too small.
  - If the market is not cleared often enough, people may waste time waiting for the next clearing date.
  - It can still work (e.g. kidney exchange) but there is a trade-off.
- This is not an issue with kindergarten
  - Everyone starts at the same time.
  - The whole market is cleared once a year.
  - Kindergarten is a good candidate for centralisation.

Towards Centralisation



- Centralised enrolment appeared in 2003.
  - In 2011, 39 out of 79 councils used central enrolment.
  - MAV published a guide in 2013 (see references).
- **Darebin**
  - The council directly manages the enrolment process of all 41 kindergartens within the municipality.
- **Bendigo**
  - The Loddon Mallee Preschool Association manages the central enrolment system.
  - Kindergartens from neighbouring councils are included.

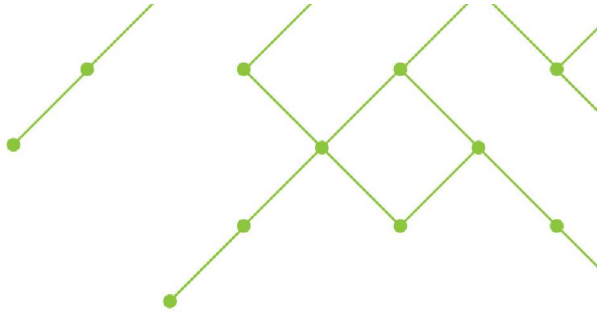
Further Improvement



- Centralisation provides many advantages.
  - One application per council means less paperwork for both sides of the market.
  - A centralised system involves less gaming for parents.
  - Councils may get a better idea of what the demand is.

- What else can be improved?
  - The matching mechanism within each centralised system is far from optimal.
  - Centralisation could be extended to the whole State.

Allocation Procedures



- **Darebin** allows families to rank up to four kindergartens and uses a four-round procedure.
  - First round offers are made in July.
  - Four weeks later, second round offers are made using available places after the first round offers.
  - Third round offers are made three weeks later.
  - Remaining vacancies are offered to unmatched children.

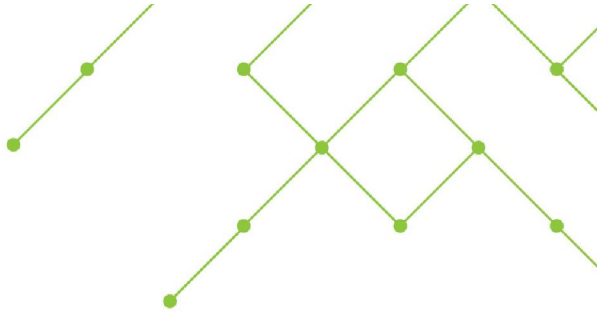
- Sounds familiar?
  - This is (almost) exactly equivalent to the IA algorithm.
  - The difference is that it is done manually over two months.

Allocation Procedures

- **Bendigo** has a similar procedure.
  - Maximum of five choices per family.
  - Six rounds of offers over two and a half months.
  - Additionally, priorities depend on preferences.

- **Shepparton** has a more idiosyncratic procedure.
  - Each family has one choice.
  - If rejected, they can choose to remain on the waiting list or apply for another kindergarten that has vacancy.
  - It is not clear how these vacancies are allocated.

Immediate Acceptance



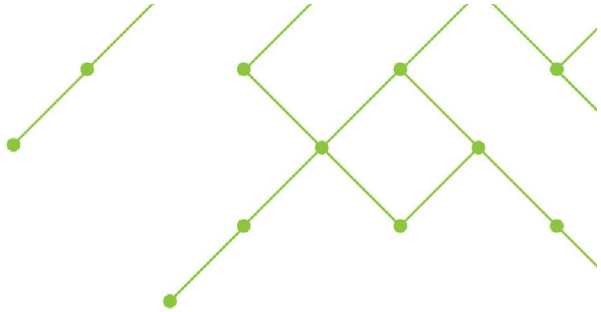
- Using the IA algorithm would improve the situation.
  - The allocation is calculated in a few seconds, not a few months.
  - The rules are clearer, easier to explain to families and consistent across councils.
  - Families can be allowed to select as many choices as they like without creating more work other than data entry.
  - The allocation improves from IA with a limited number of choices to IA without such restriction.

Lessons from School Choice

- The IA algorithm has many flaws.
  - Parents must be careful when reporting their preferences, in particular ranking a popular kindergarten first is very risky.
  - The priorities are not always respected. It is possible to lose a place to a child with a lower priority (justified envy, stability).

- There exist two better algorithms:
  - Deferred Acceptance (DA).
  - Top-Trading Cycle (TTC).

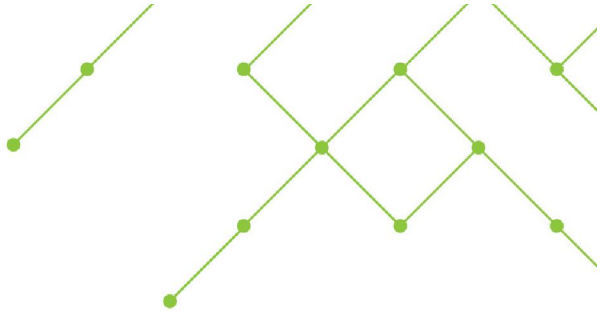
Two Algorithms



- **Deferred Acceptance**
  - The algorithm is strategy-proof, parents can never gain by misreporting their preferences.
  - The matching always respects priorities, a child can never lose a place to another one with lower priorities.
  - The matching is the most efficient one that does not violate priorities but mutually beneficial trading possibilities do exist.

- **Top-trading Cycle**
  - Strategy-proof as well.
  - The matching is Pareto-efficient, no trade is possible.
  - The matching takes priorities into account as much as possible. but violations may be necessary to achieve efficiency.

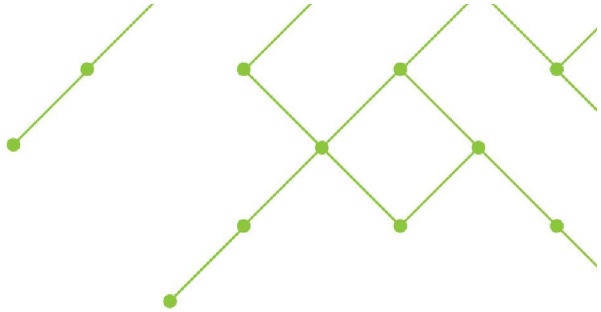
DA vs TTC



- The Boston School Committee chose the DA algorithm.
  - Priorities are given either because the student lives nearby or because (s)he has a sibling attending that school.
  - With TTC, students trade their priorities. A student can get a place at School B because of his/her priority at School A.
  - The priority at School B was given because (s)he had good reasons to want to go there.
  - Getting a place at School A because of this is not right.

- This argument is valid for kindergartens as well.
  - Getting a place in kindergarten A because a sibling goes to a school in the same building as kindergarten B is not right.

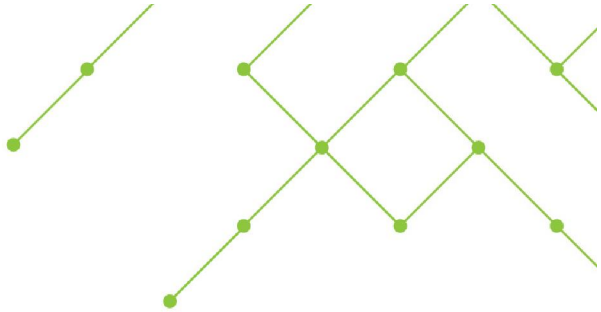
DA vs TTC (cont.)



- If TTC is used and each kindergarten only ranks its applicants.
  - Incentive to apply to non-acceptable kindergartens in order to have more priorities to trade. TTC is not strategy-proof.
  - Places may be given to families who do not want them.
  - Families who apply to many places are unfairly favoured.

- TTC requires that every child be ranked by each kindergarten.
  - Possible but can be extremely tedious in large market.
  - DA requires significantly less work.

Benefits of DA



- The CMD recommended the deferred-acceptance algorithm be used for kindergarten allocation in Victoria.
- The benefits of moving from a manual to an algorithmic matching system remain.
  - Quicker, less costly, less paperwork, better allocation.
- Moving from IA to DA yields additional benefits.
  - Parents can be clearly told they should report truthfully.
  - This allows better measuring demand and efficiently adapt capacities in future years.
  - The matching is fair, priorities are always respected.

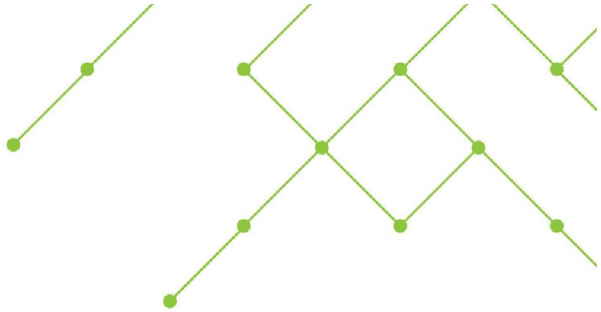
People Commute

Source: Victorian Electoral Commission Website

A Central Clearing House

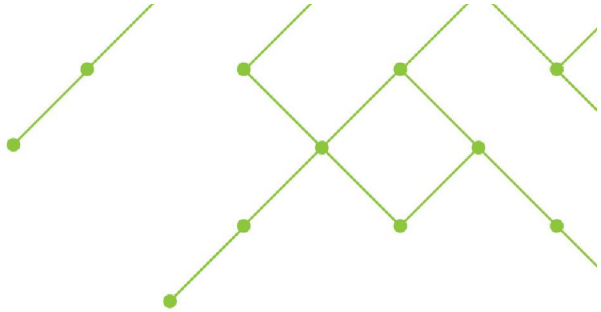
- Suppose DA is implemented in all councils.
  - Parents may wish to look at different councils.
  - They will have to apply separately to each one.
  - The inefficiencies of decentralisation reappear.
- A solution is to have a single clearing house for Victoria.
  - Difficult to do with a manual matching process, easier with an algorithmic one.
  - Each kindergarten determines its priority rules, each family fills one application form.
  - The clearing house collects this information and the computer calculates the matching.

Possible Obstacles



- People like to be in control rather than let a mysterious computer program do the job.
  - Explaining the algorithm goes a long way.
- Councils may feel power is taken away from them.
  - They would retain the same freedom to choose priorities.
  - They will still manage kindergartens as they do now.
- People do not like change.
  - Start with a pilot in one or two councils.

Possible Extensions

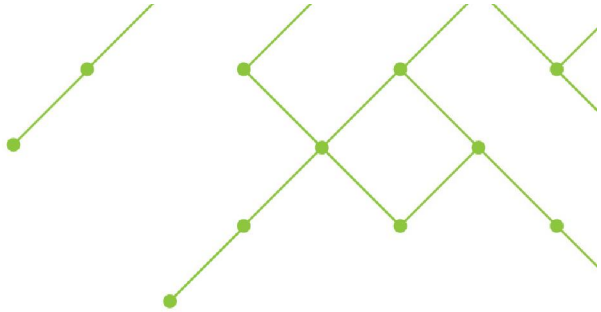


- **Schools**
  - Many American cities use a matching system.
  - The model is similar to kindergarten.
  - The potential gains are much larger.

- **Child Care**
  - Every parent know how hard it is to find a place.
  - The current system is completely decentralised.
  - Potential gains are much larger than for kindergartens.
  - Part-time makes it a much more difficult model.

DTF Job Transfer

Job Swap

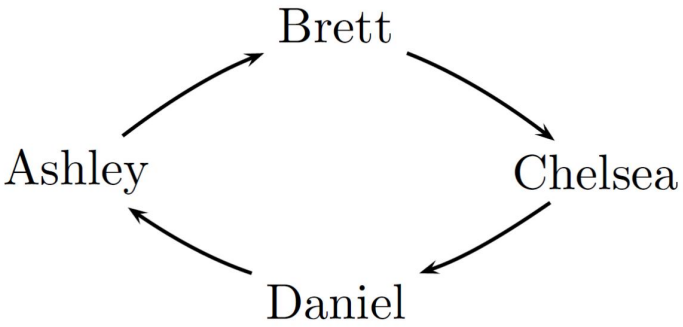


- The Department of Treasury and Finance (DTF) is an important part of the Victorian Government.
  - 19 groups.
  - 643 employees.
  - 534 full-time.

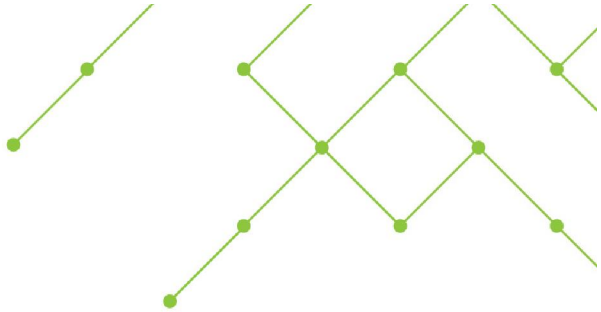
- Every few years, employees may elect to change group.
  - No position is created or deleted.
  - Employees simply swap positions.
  - Same level of hierarchy.

Current System

- A swap happens if two employees are willing to exchange their positions.
  - It requires a double occurrence.
  - Few swaps are likely to happen.
- Allowing larger cycles will be beneficial.
  - This is the idea of the TTC algorithm.

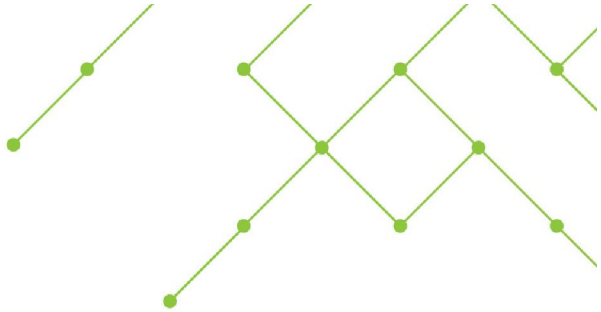


A Matching Market



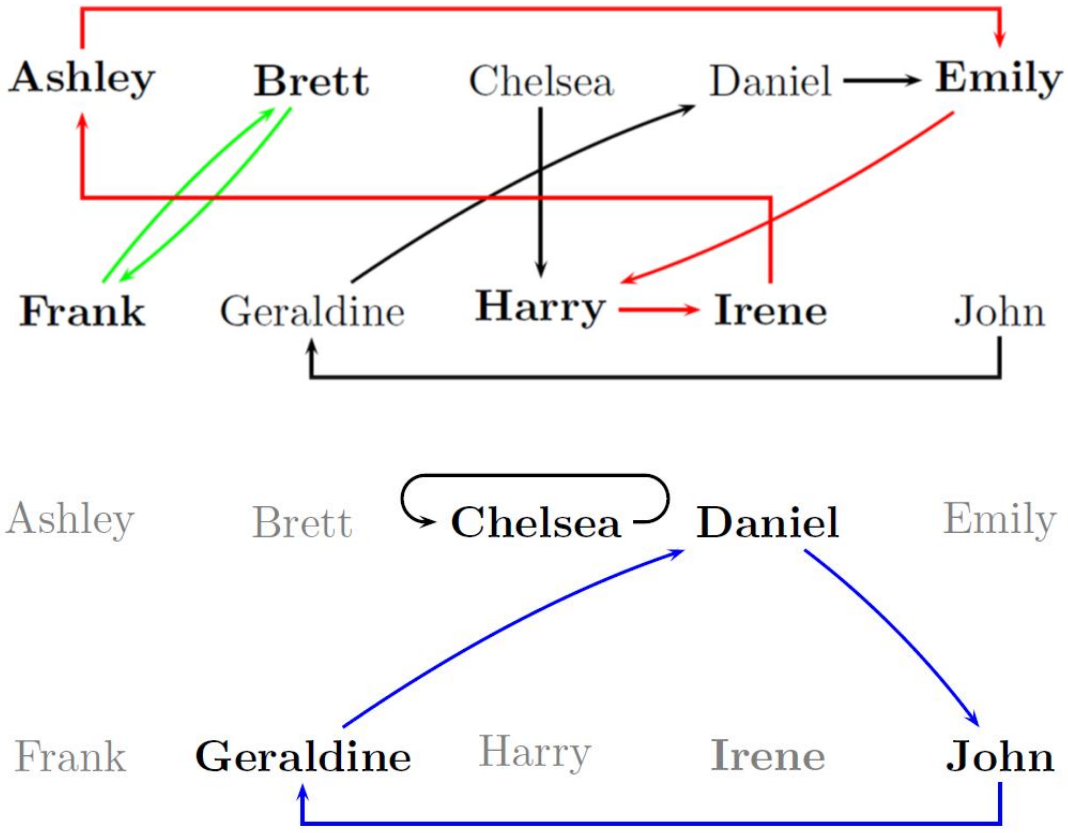
- The market is one-sided
  - Positions are not agents.
- The matching is one-to-one.
  - Same number of employees and positions.
  - The outside option is one's current position.
- Employees have preferences over positions.
  - Including their own.
- Employees have priority for their own position.
  - No other priority is needed if TTC is used.
  - A position will exit as soon as it is in a cycle.

Top-Trading Cycle



- **Process**
  - Employees who are interested sign up.
  - The list of possible positions is communicated to them.
  - Employees rank positions in order of preferences.
  - No need to rank positions beyond their own.
  - The matching is determined by the TTC algorithm.
- **Properties**
  - Employees have no incentive to misreport.
  - The matching is Pareto-efficient, all possible gains from trade have been achieved.
  - Employees do not risk their position by signing up, they will only move if they can have a position they prefer.

TTC at Work


Ashley
Brett
Chelsea
Daniel
Emily
Frank
Geraldine
Harry
Irene
John
Ashley
Brett
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Harry
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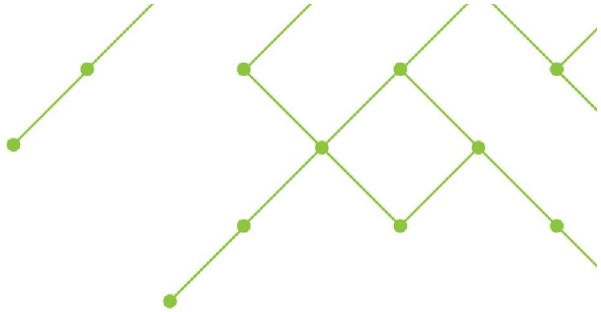
John

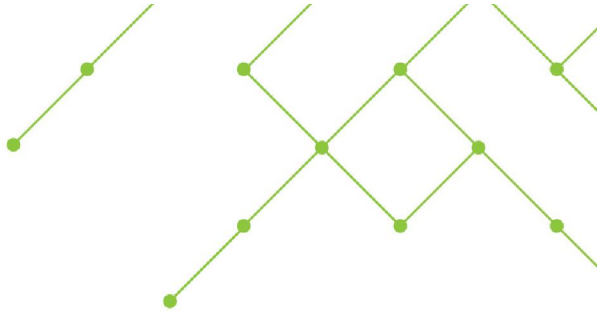
Director Preferences



- Directors are likely to care about who joins their group.
  - They should be able to have a say.
  - The market becomes two-sided as directors are agents.
  - TTC does not perform well in two-sided markets.
  - DA becomes the natural choice.

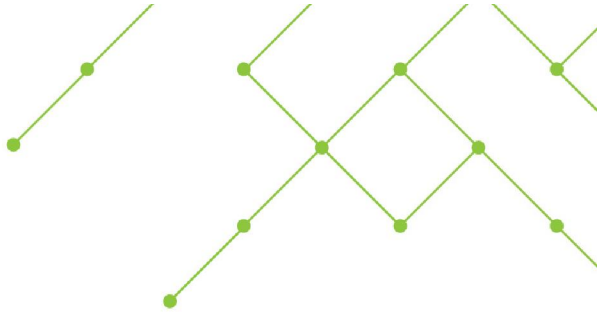
- Employees still have preferences over positions.
  - Directors now have preferences over employees who applied for positions within their group.
  - The matching is still one-to-one.

Deferred-Acceptance



- Two versions of DA can be considered.
  - Employee-proposing or Director-proposing.
- Employee-proposing DA is the natural choice.
  - The program is designed for employees.
  - Directors may manage more than one position.
- **Properties**
  - The matching is stable.
  - It is the best stable matching for employees.
  - Employees cannot gain by misreporting.
  - Directors are unlikely to gain by misreporting.

Risk of Signing Up



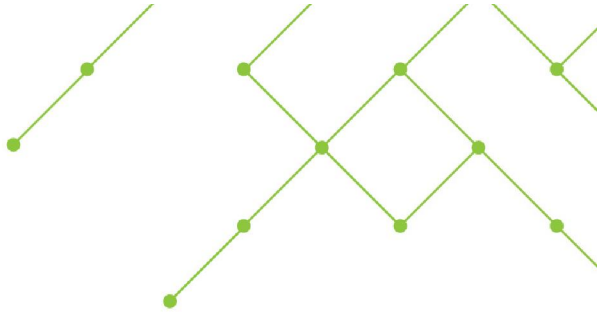
- Employees may regret having signed up.
  - They can receive a position they like less than their own.
  - Possible if their directors did not rank them first.
  - Signing up is risky for employees.
  - This might deter them from participating.
  - Against the goal and the spirit of this program.

- **Solution**
  - Employees are given first priority for their own position.
  - The remaining priorities depend on director preferences.
  - Employees will at worse stay in their current position.

Final Design

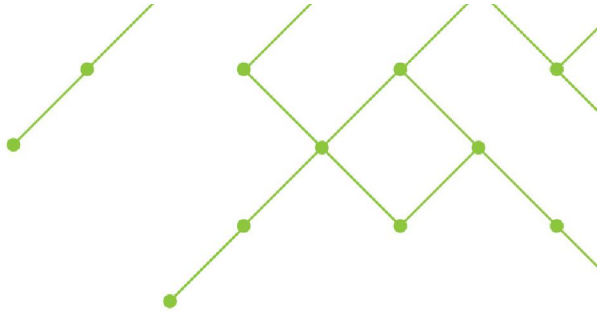
- **Process**
  - Employees sign-up for the program.
  - The list of positions is made available to them.
  - Employees select and rank those they would like.
  - For each position within their group, directors rank applicants in order of preferences.
  - For each position, the current employee gets first priority, the other priorities are chosen by the group director.
  - The employee-proposing DA algorithm selects the matching based on employee preferences and priorities.

Final Design



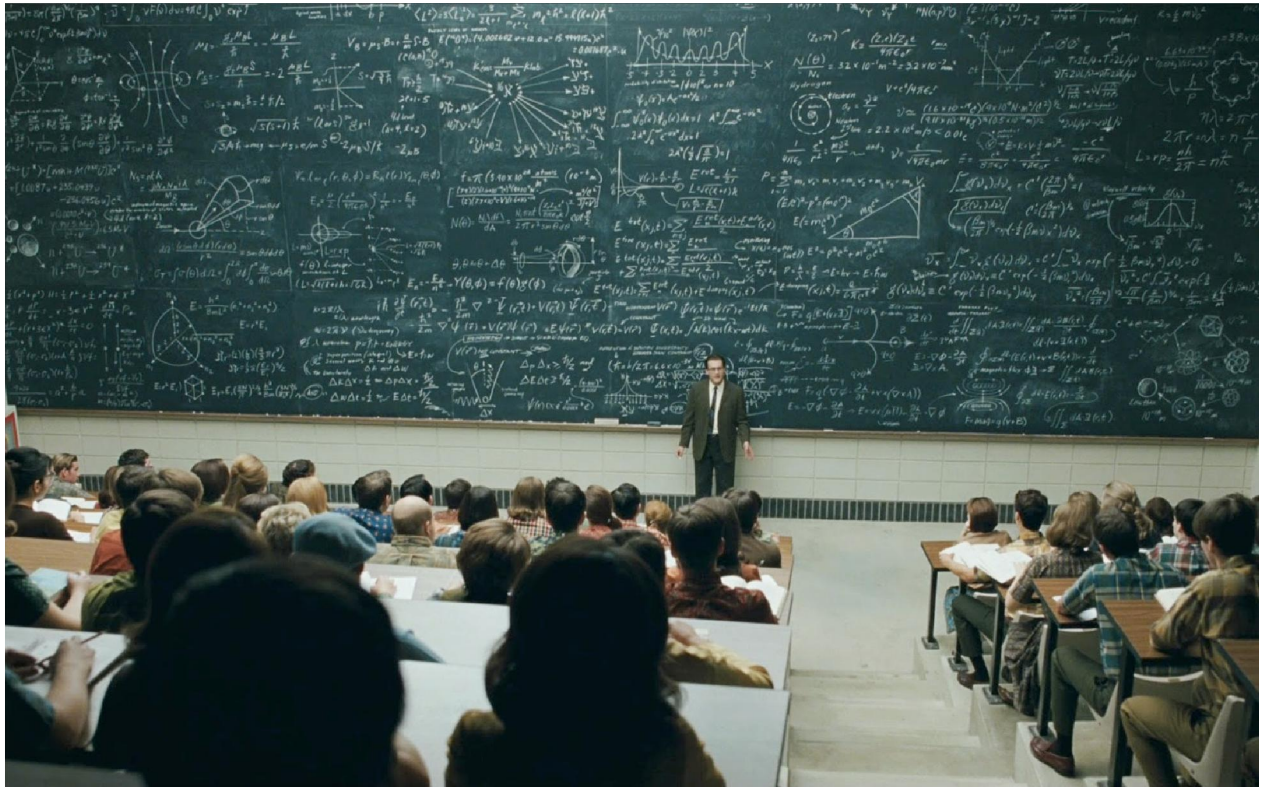
- **Properties**
  - Employees do not take any risk by signing up.
  - Employees can never gain by misreporting.
  - Priorities are always respected (stable matching).
  - The matching is the best possible for employees that does not violate priorities. It is not as good for directors.
  - Directors preferences are taken into account but only if their current employee leaves.
  - Directors may theoretically gain by misreporting but they are much more likely to lose if they do so.

Trade-Off

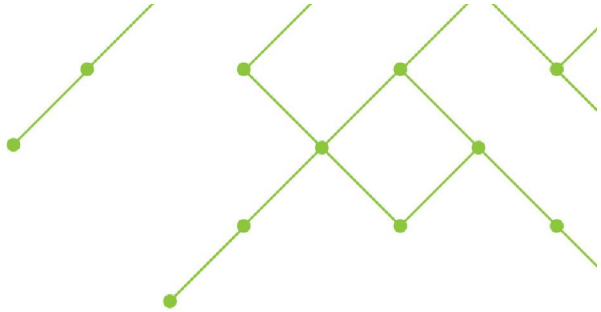


- **Advantage of TTC**
  - Efficient matching, gains from trade are maximised.
- **Advantage of DA**
  - Directors preferences are taken into account.
- **Discussion**
  - Directors preferences are imperfectly taken into account as they do not rank the employee currently holding the position and DA is used with employee proposing .
  - The new design sacrifices as little efficiency as possible in order to give directors a say. It keeps all other properties.
  - Both designs have desirable properties and constitute an important improvement over the current system.

Tertiary Education in Victoria

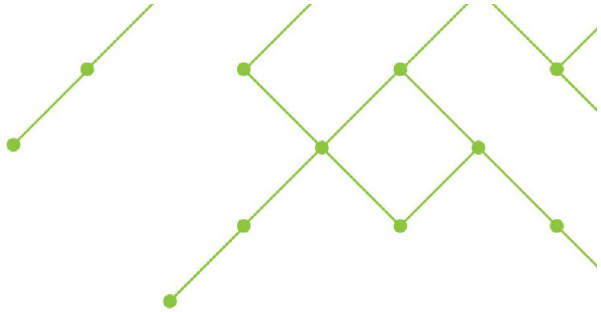


A Matching Problem



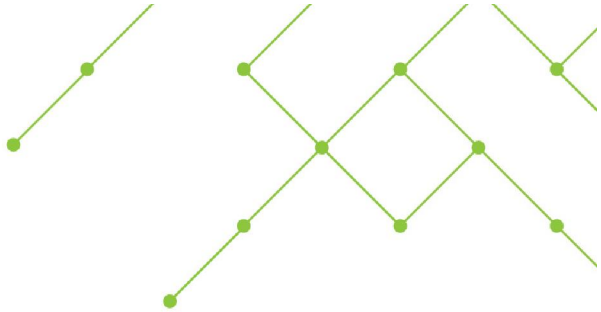
- Two-sided market
  - Courses are offered by institutions.
  - Institutions are strategic agents.
- Many-to-one matching
  - One course per student, many students per course.
  - Places are limited, demand exceeds supply.
- Student preferences
  - Discipline that interests them.
  - Best institution.
- Institution preferences
  - Institutions compete for the best students.

Decentralised System



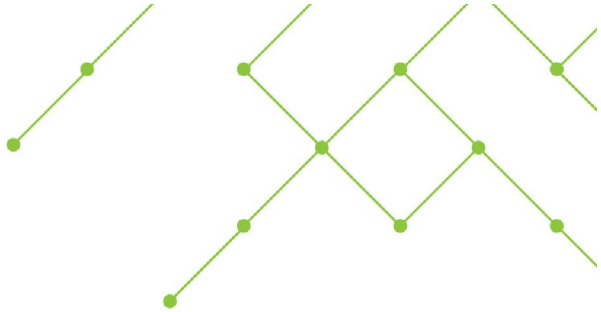
- **Coordination problem**
  - Same issue as for kindergartens.
  - Students apply directly to institutions.
  - Institutions make an offer to their best applicants.
  - Students accept their best offer.
  - Institutions get extra capacity and can make new offers.
- **Unravelling**
  - Only happens in two-sided markets.
  - Problem faced by medical graduates in the US and UK.
  - Waiting is risky, good matches may be gone.
  - Incentive to commit early.
  - Lack of information, flexibility and time to decide.

College Admission



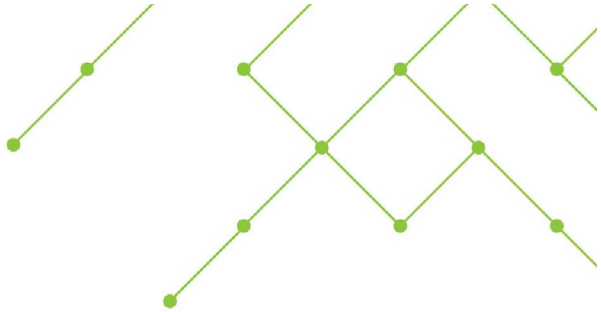
- College Admission is the first matching model ever studied.
  - Gale and Shapley (1962).
- **Solution (GS 1962)**
  - Students rank acceptable colleges.
  - Colleges rank acceptable students and set their capacity.
  - The student-proposing DA algorithm determines the matching.
- **Properties**
  - No coordination problem.
  - The matching is stable.
  - Students cannot gain by misreporting.
  - Colleges are unlikely to gain by misreporting.

VTAC



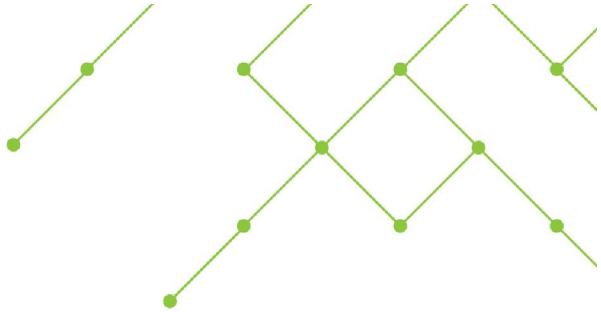
- Applications are managed centrally by the **Victorian Tertiary Admission Centre** (VTAC).
  - www.vtac.edu.au
- VTAC manages applications for courses provided by 65 institutions throughout the State.
  - 12 universities.
  - 19 TAFE institutes.
  - 34 independent tertiary colleges.
- VTAC's duties also include
  - Calculate and communicate ATAR scores.
  - Manage scholarship applications.

Theory vs Reality



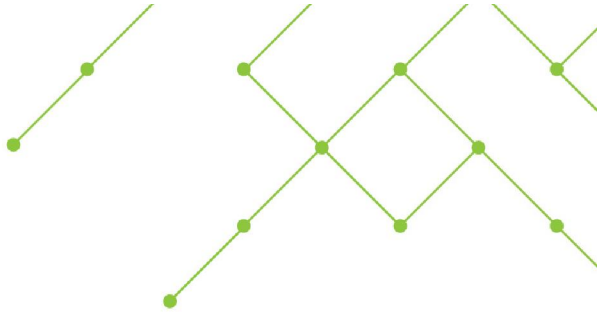
- Ranking all courses is hard for students.
  - Large number of possibilities.
  - Researching is costly.
  - A change of mind is possible.
- Ranking all students is hard for institutions.
  - Large number of students.
  - Interviewing them all is unrealistic.
- Institutions care about both quantity and quality.
  - Optimal cohort size depends on the quality of applicants.

Student Preferences



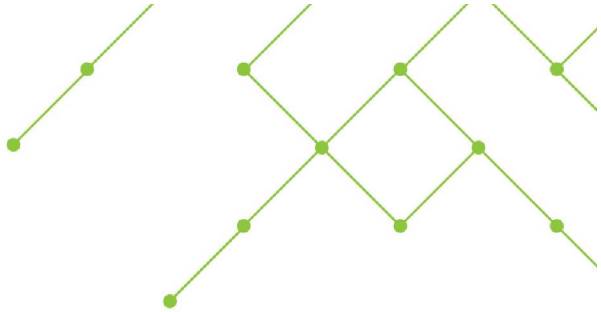
- The number of choices per student is limited to 12.
  - Truthful reporting may become risky.
  - Why is it so?
- One explanation might be logistic.
  - More choices means more work for VTAC.
  - However, preferences are entered online and the matching is calculated by a computer.
- Another explanation may be psychological.
  - Given full freedom, students may list too few choices.
  - If there is a limited number of choices, they may feel like they should use all of them.

Institution Preferences



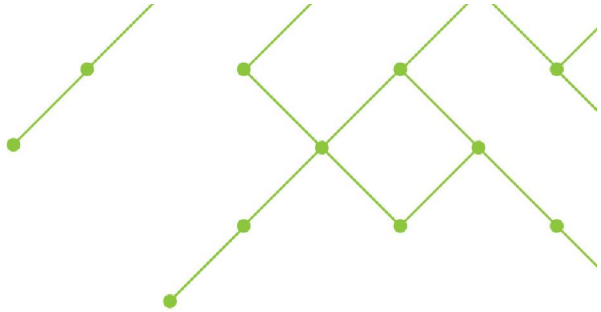
- Institutions can reveal their preferences in two ways:
  - Rank students in order of preference.
  - Choose an acceptability cut-off.
- There is a capacity associated with each course.
  - Not as strict as for kindergarten, accommodating a few more students is often feasible.
  - It can be manipulated. Institutions can choose to accept at most 50 students even though they could technically accommodate 100.
- Institution preferences are complex
  - The model is too restricted.

Ranking Students



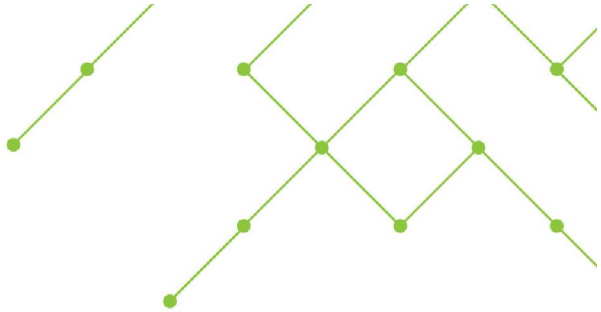
- Ranking students is difficult.
  - Large number of students.
  - Limited information about them.
- Australian Tertiary Admission Rank (ATAR)
  - In Victoria, based on VCE results.
  - Increment of 0.05, best possible score is 99.95.
- ATAR score can be combined with other criteria.
  - For example, some programs require students to add a personal statement to their VTAC application.
  - Institution are free to combine different criteria or even choose their own ranking. ATAR is often used in practice.

Preferences over Cohorts



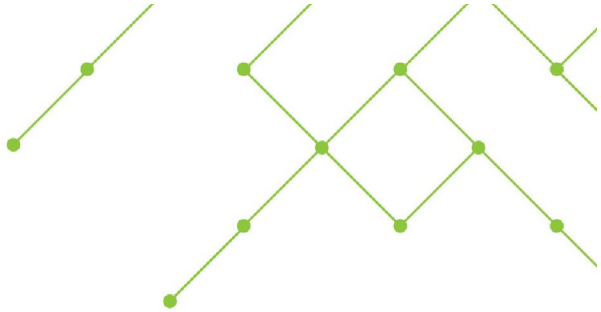
- Institutions care about both quantity and quality.
  - More students means more tuition fees.
  - Better students means better reputation.
- Optimal cohort depends on a trade-off.
  - Large intake with low average quality.
  - Small intake with high quality.
- Institution preferences are complex
  - Preferences over each possible cohort.
  - *U*(*size*, *quality*).
  - The model does not account for this.

Rejecting Students



- Institutions have two instruments to reject students.
  - Set a capacity constraint, e.g. max 50 students.
  - Set a cut-off, e.g. no student below 75.
  - They can combine both but one will be redundant.
- Example where the capacity is binding.
  - 50 students, lowest score is 80.
  - Cut-off is redundant.
- Example where the cut-off is binding.
  - 40 students, lowest score must be 75.
  - Capacity constraint is redundant.

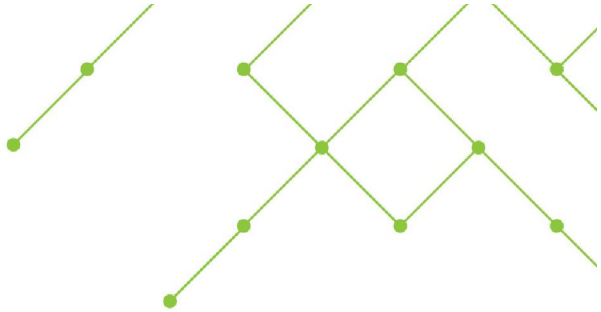
Example



- Institution chooses the following rejection strategy.
  - No more than 50 students.
  - No student below 75.

- This strategy may prove too lenient.
  - Accepted cohort has 50 students, lowest score is 76.
  - 5 of the students have a score between 76 and 82.
  - Could have set capacity to 45 or cut-off to 82.
  - Quite possibly $U(45,82) > U(50,76)$.

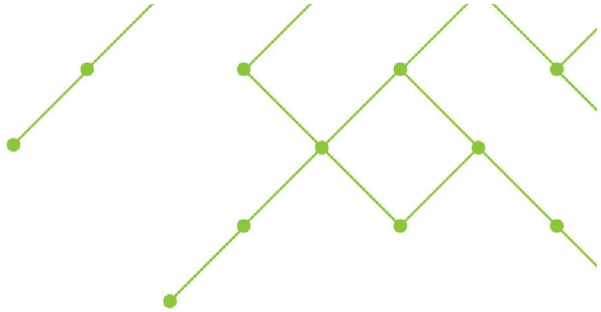
Example (cont.)



- Capacity may be too tough.
  - Get 50 students with lowest score 80.
  - Rejected 10 students with score between 79 and 80.
  - Could have set capacity to 60 instead of 50.
  - Quite possibly, $U(60,79) > U(50,80)$.

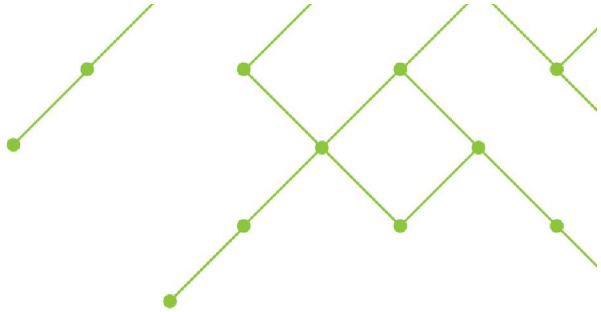
- Cut-off may be too tough
  - Get 20 students with lowest score 75.
  - Rejected 20 students with score between 71 and 75.
  - Could have set cut-off to 71 instead of 75.
  - Quite possibly, $U(40,71) > U(20,75)$.

Information Problem



- Perfect information.
  - Need only one instrument to achieve the optimal cohort.
- Imperfect information.
  - Depending on demand, the same policy can yield either too much quantity or too much quality.
- Estimating demand is complex.
  - It depends on student pref. and institution strategies.
  - It is difficult for institutions to get it right.
- The problem is serious.
  - Institutions can leave the program if not satisfied.

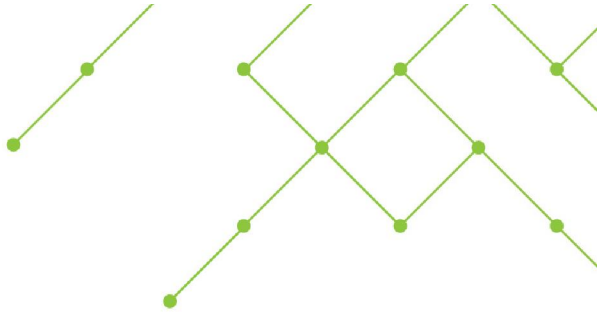
Solution



- Ask universities to rank every cohort.
  - Ranking every students is hard enough.
  - Cannot run the DA algorithm with such preferences.
  - This solution is not realistic.

- Give institutions a chance to learn about their demand.
  - Institutions can then choose an appropriate policy.
  - This is what VTAC does.
  - How does it work?

Implementation



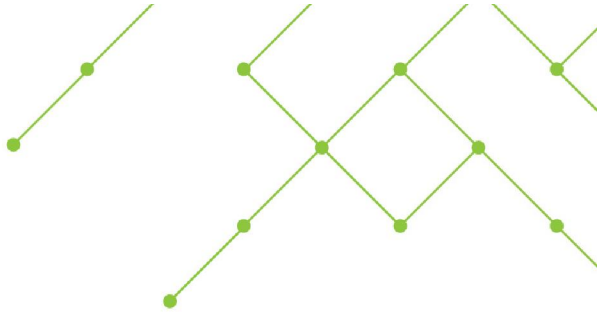
- VTAC runs non-binding trials
  - Institutions choose a policy and DA is run.
  - Institutions observe their cohort.
  - They can change their policy in the next trial.
  - After enough trials, they know their demand well.

- Rejection policy
  - Institutions learn about demand for their courses.
  - They only need one instrument.
  - Typically, they will simply choose a cut-off.

Process

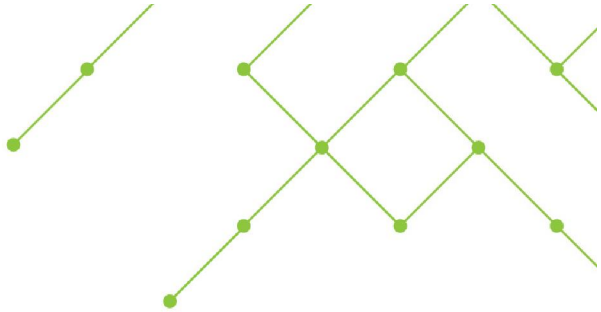
- Students submit their preferences.
  - They learn their ATAR score before the deadline.
- Institutions choose their cut-off score.
  - Some may choose a capacity or a combination of both.
- The DA algorithm is run, the outcome is not binding.
  - First trial starts after final student preferences are submitted.
  - Institutions observe their tentative cohort.
  - Students do not observe the outcome.

Process (cont.)



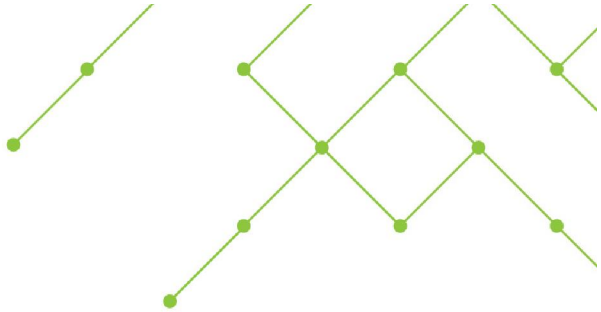
- Each institution adapts its cut-off for the next trial.
  - Lower (higher) cut-off if less (more) students than expected.
  - This affects other institutions' demand.
- The binding allocation is calculated after the trials.
  - In VIC: 2-3 trials a week before.
  - In NSW: much more trials over a few weeks.
- Each student receives at most one offer.
  - They can reject that offer but not get another one.

Properties



- Properties of DA in College Admission model.
  - The best strategy for students is to report truthfully.
  - Colleges may gain by misreporting but are more likely to lose.
  - The matching is stable.
  - It is the best possible matching for students.
- The College Admission model is too restricted.
  - Institution preferences are over cohorts.
  - They have limited information about demand.
  - Their optimal strategy is not clear.
  - The process has been adapted, what are its properties?

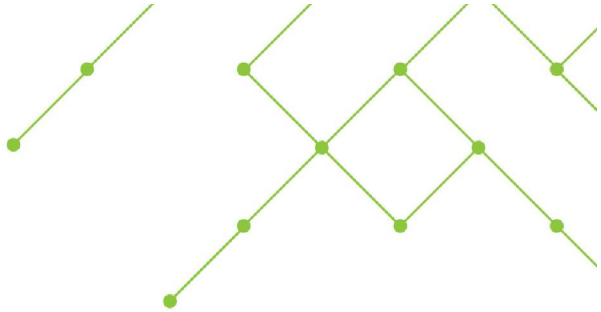
Properties (cont.)



- **Students Incentive**
  - Students are limited to 12 choices.
  - Reporting truthfully is best if 12 acceptable courses or less.
  - Otherwise may gain by dropping some choices.
  - It is still best to rank courses truthfully.

- **Institutions incentive**
  - Institutions have to play a game.
  - They are better equipped than students for that.
  - The game is simple enough to work well.

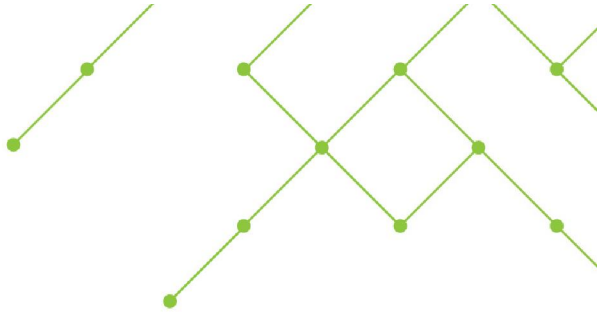
Properties (cont.)



- **Stability**
  - The matching is stable with respect to reported preferences.
  - Students may unfairly miss out on a course (s)he dropped.
  - The ranking of students by institutions is not perfect.

- **Best stable matching for students.**
  - True with respect to reported preferences.
  - Reservations regarding preference revelations remain.

Conclusion



- Matching theory is relatively new
  - The literature started in 1962 (Gale-Shapley) but has really taken off in the past 10-15 years.
- The number of possible applications is large.
  - Some fit the model well and are “easy” to implement.
  - Some are more complex (e.g. childcare).
- The use of matching theory in Australia is limited.
  - University entry and kidney exchange.
  - Low hanging fruits are still there to be picked.
  - It is important that public servants develop a basic understanding of matching theory.

References

- Municipality Association of Victoria, Jan 2013. “A Framework and Resource Guide for Managing a Central Registrations Process for Kindergarten Places”. Available here.

- Victorian Government, Department of Education, Mar 2014. “The Kindergarten Guide 2014”. Available here.

Kindergarten Information

- Bendigo
  - www.lmpa.org.au Information Booklet Application Form
- Boroondara
  - Website Enrolment Policy Application Form
- Darebin
  - Website
- Monash
  - Website Kindergarten Guide Application Form
- Shepparton
  - Website Application Form