Matching Mechanisms for Refugee Resettlement

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Guest Lecture
AS.180.244 Market Design
AS.180.645 Topics in Economic Theory
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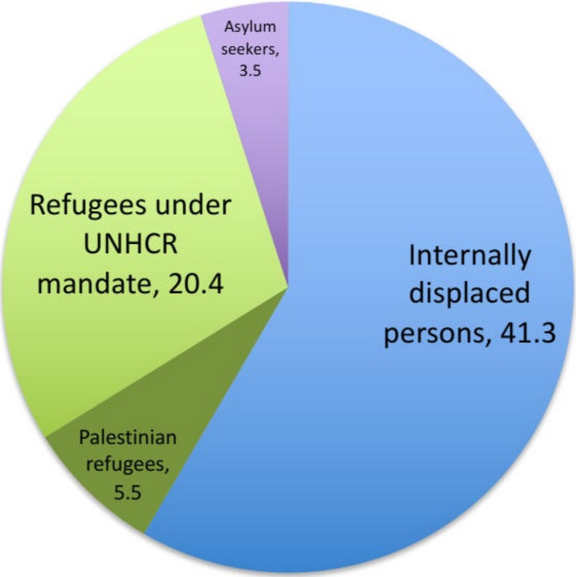
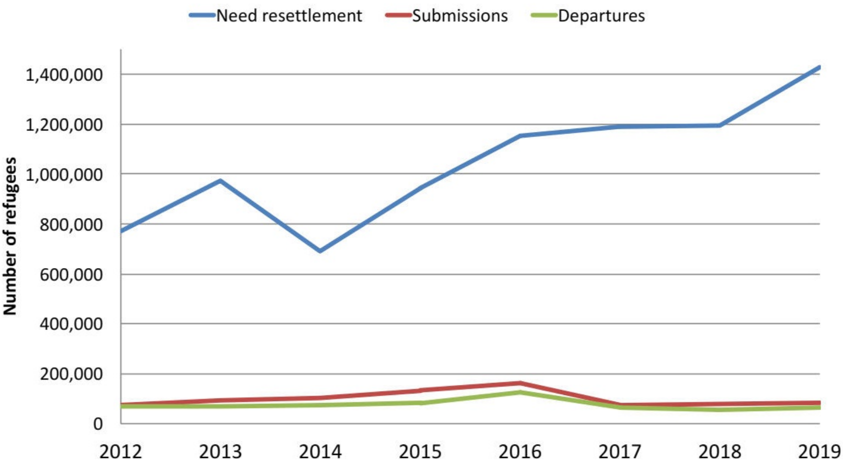
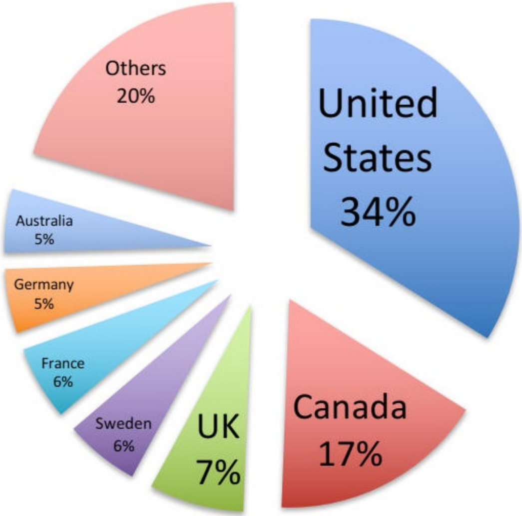
70M Refugees around the World



Resettlement Needs, Submissions, and Departures



Resettlement Destinations (2018)



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- One of them is the Hebrew Immigrant Aid Society (HIAS)

The Matching Problem

Each resettlement agency has links to localities

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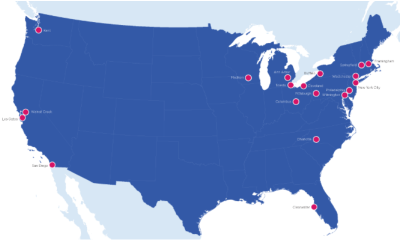

Figure: HIAS' network of localities.

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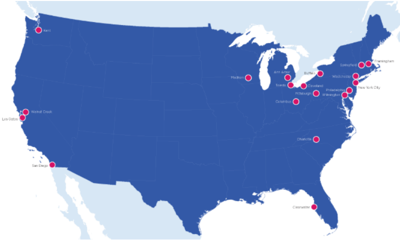

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How to best match refugees and localities?

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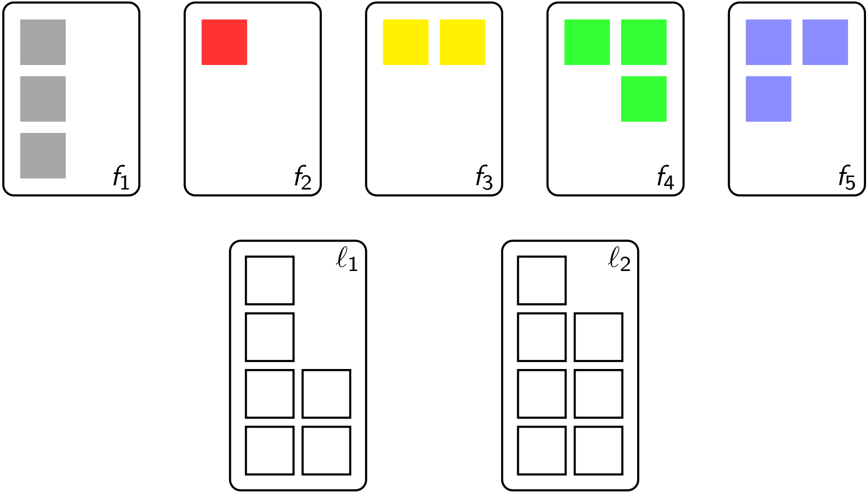
Matching Market with Multidimensional Constraints

Multidimensional Constraints



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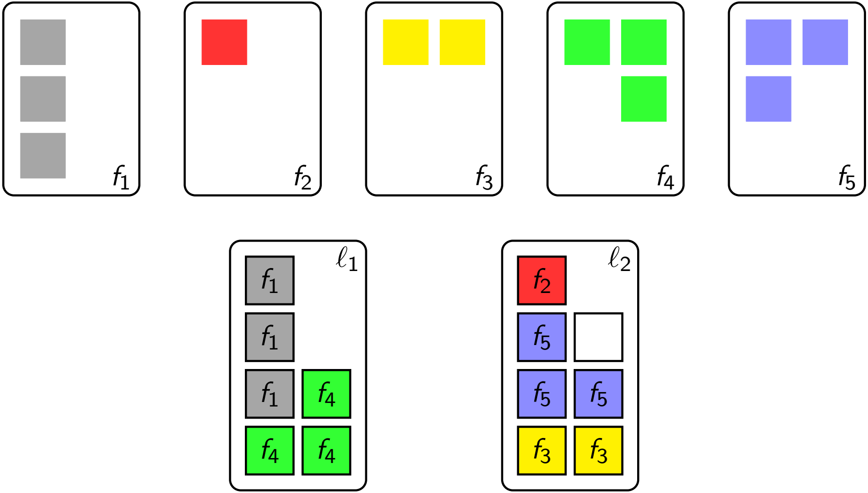

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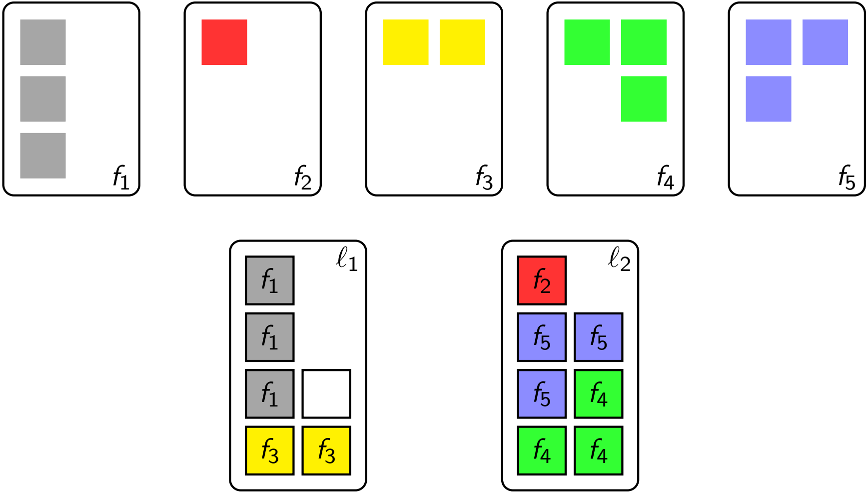

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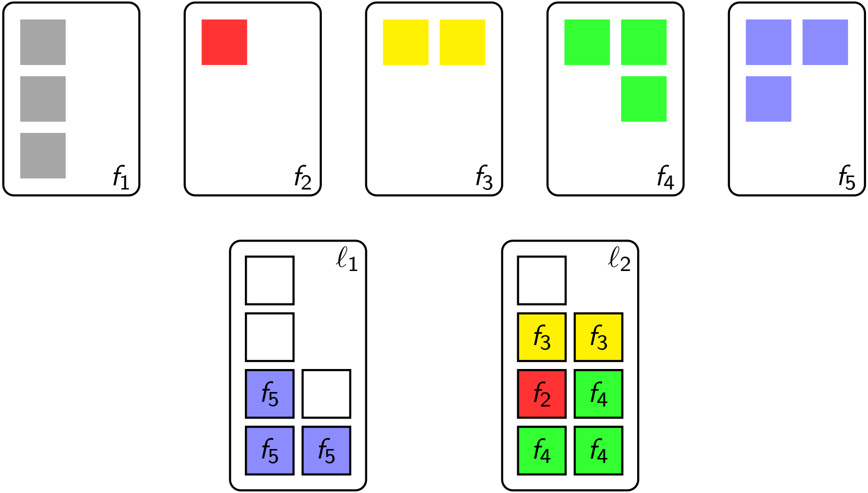

Static vs Dynamic Matching

Capacity constraints are set once a year

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We consider the static problem

- We take capacities as given and treat them as hard constraints
- Dynamic capacity management would constitute a valuable extension

Matching on Observables

1 May 2018: For the first time, a US resettlement agency used a software to algorithmically match refugees

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- Other resettlement agencies do the matching by hand

Contribution

We propose to take preferences into account

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"Many Somali refugees initially settled around the country subsequently migrated to Lewiston, Maine. Lewiston has a weak economy but an established Somali community. Consequently, efforts to resettle these refugees elsewhere in the U.S. were less effective than they could have been. Their preferences should have been taken into account from the start."
— Mark Hetfield (CEO of HIAS) in Roth (2015), "Migrants aren't widgets", Politico

Model

Set of families F, set of localities L

- Families have strict and ordinal preferences over localities

$$\succ_f : \ell_1, \ell_2, \ell_3, \ldots$$

- Each locality strictly ranks families in order of priority

$$\triangleright_\ell : f_1, f_2, f_3, \ldots$$

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Set of services S (# of refugees, school places, medical needs, etc)

- Family f requires $\nu_s^f \in \mathbb{Z}_{\geq 0}$ units of service s
- Locality ℓ can provide $\kappa_s^\ell \in \mathbb{Z}_{\geq 0}$ units of service s

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- Satisfies all multidimensional constraints:

$$\sum_{f \in \mu(\ell)} \nu_s^f \leq \kappa_s^\ell \quad \text{for all} \quad \ell \in L \text{ and } s \in S$$

Plan

Part I: Improve upon an endowment

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- Mechanism: Multidimensional Top Trading Cycles with Endowment

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We use a modified version of Top Trading Cycles

Top Trading Cycles in School Choice

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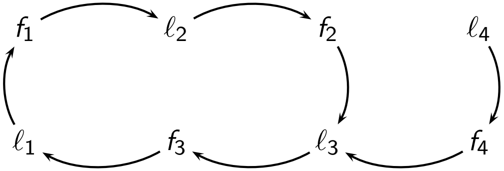

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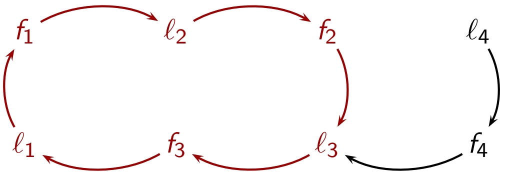

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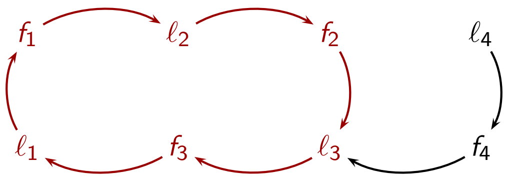

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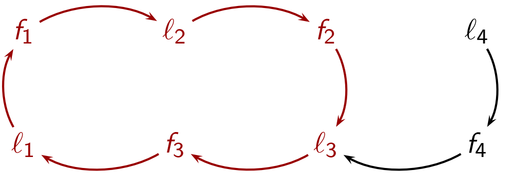
- A priority can be interpreted as an endowment



Top Trading Cycles in Refugee Resettlement

TTC can easily be adapted to our enviroment

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Improving upon an endowment is challenging

- Families of different sizes may not be able to swap with each other
- This problem does not occur in school choice

Pareto-improving Chains

Identifying Pareto-improvements is challenging

- Families with different needs may have to move simultaneously
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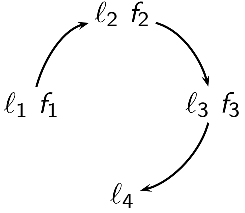

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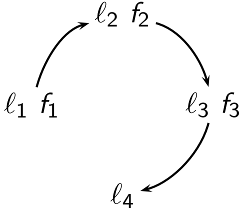

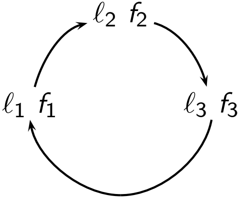

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Theorem

There does not exist any individually rational, chain-efficient, and strategy-proof mechanism.

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Multidimensional Top-Trading Cycles with Endowment (MTTCE).

- Identifies and carries out chains to improve upon the endowment
- Individually rational and strategy-proof
- Pareto-improves upon any chain-inefficient endowment when $|S| = 1$

Plan

- Part I: Improve upon an endowment
  - Start with a matching (e.g., matching on observables)
  - Use refugee preferences to find Pareto improvements
  - Mechanism: Multidimensional Top Trading Cycles with Endowment

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How to account for both preferences and priorities?

- Stability is the natural solution concept

Example from McDermid and Manlove (2010)

Preferences

$f_1 : \ell_2, \ell_1, \emptyset$

$f_2 : \ell_1, \ell_2, \emptyset$

$f_3 : \ell_1, \emptyset$

Priorities

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Round 1		
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f_3	$\rightarrow$	ℓ_1

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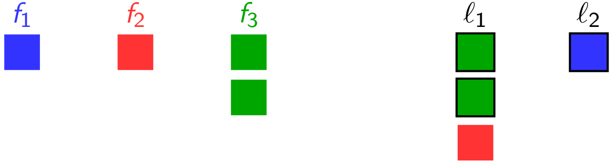

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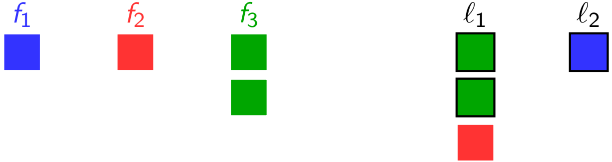

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$f_3 : \ell_1, \emptyset$

Priorities

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$\ell_2 : f_2, f_1, f_3$

Round 1			
f_1	$\rightarrow$	ℓ_2	✓
f_2	$\rightarrow$	ℓ_1	✗
f_3	$\rightarrow$	ℓ_1	✓

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Round 1				Round 2		
f_1	$\rightarrow$	ℓ_2	✓	f_1	$\rightarrow$	ℓ_2
f_2	$\rightarrow$	ℓ_1	✗	f_2	$\rightarrow$	ℓ_2
f_3	$\rightarrow$	ℓ_1	✓	f_3	$\rightarrow$	ℓ_1

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Round 1				Round 2		
f_1	$\rightarrow$	ℓ_2	✓	f_1	$\rightarrow$	ℓ_2
f_2	$\rightarrow$	ℓ_1	✗	f_2	$\rightarrow$	ℓ_2
f_3	$\rightarrow$	ℓ_1	✓	f_3	$\rightarrow$	ℓ_1

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Round 1				Round 2			
f_1	$\rightarrow$	ℓ_2	✓	f_1	$\rightarrow$	ℓ_2	✗
f_2	$\rightarrow$	ℓ_1	✗	f_2	$\rightarrow$	ℓ_2	✓
f_3	$\rightarrow$	ℓ_1	✓	f_3	$\rightarrow$	ℓ_1	✓

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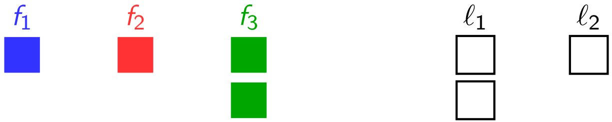

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Round 1				Round 2				Round 3		
f_1	$\rightarrow$	ℓ_2	✓	f_1	$\rightarrow$	ℓ_2	✗	f_1	$\rightarrow$	ℓ_1
f_2	$\rightarrow$	ℓ_1	✗	f_2	$\rightarrow$	ℓ_2	✓	f_2	$\rightarrow$	ℓ_2
f_3	$\rightarrow$	ℓ_1	✓	f_3	$\rightarrow$	ℓ_1	✓	f_3	$\rightarrow$	ℓ_1

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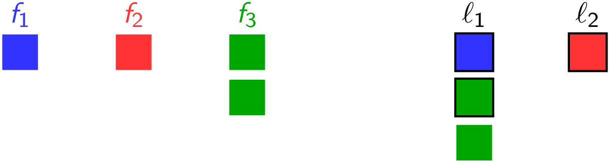

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Round 1				Round 2				Round 3		
f_1	$\to$	ℓ_2	✓	f_1	$\to$	ℓ_2	✗	f_1	$\to$	ℓ_1
f_2	$\to$	ℓ_1	✗	f_2	$\to$	ℓ_2	✓	f_2	$\to$	ℓ_2
f_3	$\to$	ℓ_1	✓	f_3	$\to$	ℓ_1	✓	f_3	$\to$	ℓ_1

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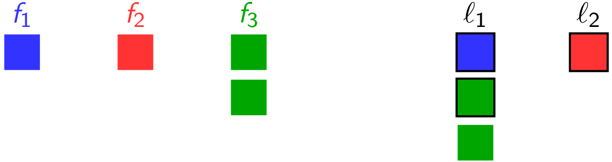

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f_1	$\rightarrow$	ℓ_2	✓	f_1	$\rightarrow$	ℓ_2	✗	f_1	$\rightarrow$	ℓ_1	✓
f_2	$\rightarrow$	ℓ_1	✗	f_2	$\rightarrow$	ℓ_2	✓	f_2	$\rightarrow$	ℓ_2	✓
f_3	$\rightarrow$	ℓ_1	✓	f_3	$\rightarrow$	ℓ_1	✓	f_3	$\rightarrow$	ℓ_1	✗

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Round 1				Round 2				Round 3				Round 4		
f_1	$\rightarrow$	ℓ_2	✓	f_1	$\rightarrow$	ℓ_2	✗	f_1	$\rightarrow$	ℓ_1	✓	f_1	$\rightarrow$	ℓ_1
f_2	$\rightarrow$	ℓ_1	✗	f_2	$\rightarrow$	ℓ_2	✓	f_2	$\rightarrow$	ℓ_2	✓	f_2	$\rightarrow$	ℓ_2
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Round 1				Round 2				Round 3				Round 4			
f_1	$\rightarrow$	ℓ_2	✓	f_1	$\rightarrow$	ℓ_2	✗	f_1	$\rightarrow$	ℓ_1	✓	f_1	$\rightarrow$	ℓ_1	
f_2	$\rightarrow$	ℓ_1	✗	f_2	$\rightarrow$	ℓ_2	✓	f_2	$\rightarrow$	ℓ_2	✓	f_2	$\rightarrow$	ℓ_2	
f_3	$\rightarrow$	ℓ_1	✓	f_3	$\rightarrow$	ℓ_1	✓	f_3	$\rightarrow$	ℓ_1	✗	f_3	$\rightarrow$	$\emptyset$	

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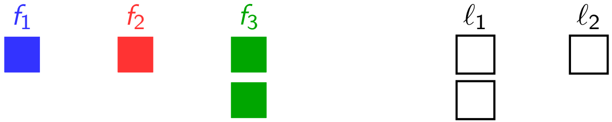
Round 1				Round 2				Round 3				Round 4			
f_1	$\rightarrow$	ℓ_2	✓	f_1	$\rightarrow$	ℓ_2	✗	f_1	$\rightarrow$	ℓ_1	✓	f_1	$\rightarrow$	ℓ_1	✓
f_2	$\rightarrow$	ℓ_1	✗	f_2	$\rightarrow$	ℓ_2	✓	f_2	$\rightarrow$	ℓ_2	✓	f_2	$\rightarrow$	ℓ_2	✓
f_3	$\rightarrow$	ℓ_1	✓	f_3	$\rightarrow$	ℓ_1	✓	f_3	$\rightarrow$	ℓ_1	✗	f_3	$\rightarrow$	$\emptyset$	✓

Example from McDermid and Manlove (2010)



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If f_3 is matched to ℓ_1

- (f_2, ℓ_2) is a blocking pair unless f_2 is matched to ℓ_2

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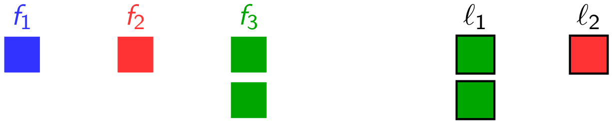

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If f_3 is matched to ℓ_1

- (f_2, ℓ_2) is a blocking pair unless f_2 is matched to ℓ_2
- But then f_1 remains unmatched and (f_1, ℓ_1) is a blocking pair

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- (f_2, ℓ_2) is a blocking pair unless f_2 is matched to ℓ_2
- But then f_1 remains unmatched and (f_1, ℓ_1) is a blocking pair
- Therefore, f_3 is not matched to ℓ_1 in any stable matching

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If f_3 is *not* matched to ℓ_1

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If f_3 is *not* matched to ℓ_1

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- (f_1, ℓ_2) is a blocking pair unless f_1 is matched to ℓ_2

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- (f_2, ℓ_1) is a blocking pair unless f_2 is matched to ℓ_1
- (f_1, ℓ_2) is a blocking pair unless f_1 is matched to ℓ_2
- But then (f_3, ℓ_1) is a blocking pair

Respecting Priorities

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Goodwill from localities matters

- They provide services
- Respecting priorities can increase their willingness to participate

How to account for both preferences and priorities?

- Stability is the natural solution concept

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Stable matchings are not guaranteed to exist

Underusing capacities may be tolerable in refugee resettlement

- We propose a solution concept that respects priorities but may underuse some capacity

(Weak) Envy-freeness

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Given a matching μ, family g envies family f if g prefers f's locality to its own and has a higher priority for it:

$$\mu(f) \succ_g \mu(g) \quad \text{and} \quad g \rhd_{\mu(f)} f$$

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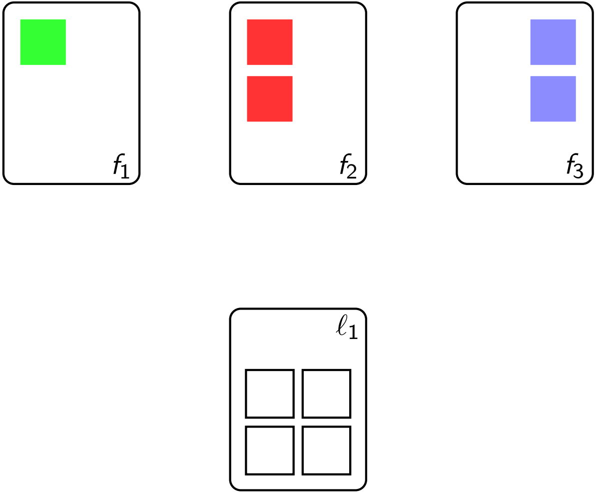
A matching is envy-free if no family envies another family

- Envy-freeness respects priorities but may underuse capacity

Weak envy-freeness is a relaxation

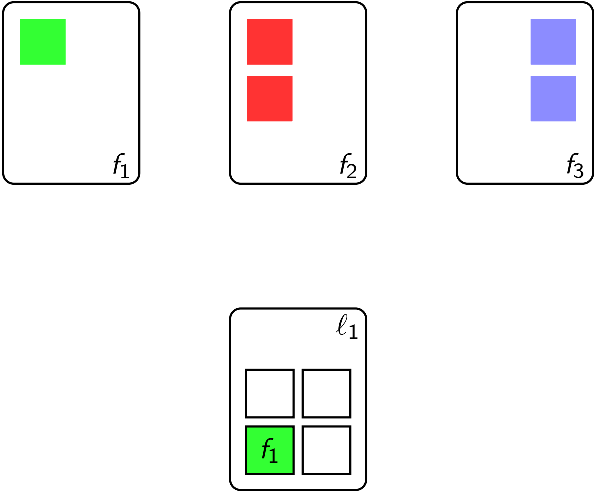
- A family f can be envied if it "fits" even when all families that envy f are matched to $\mu(f)$

Weak Envy-freeness



Priority: $f_1 \triangleright f_2 \triangleright f_3$

Weak Envy-freeness



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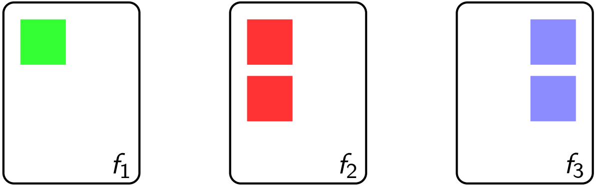
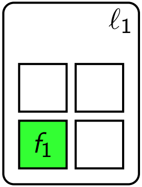
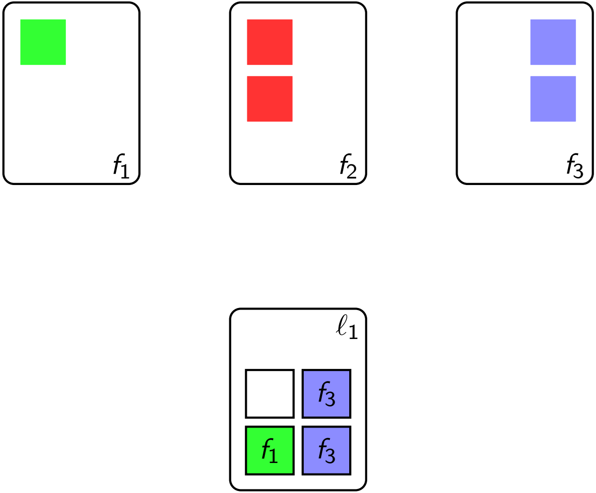


Envy-free and weakly envy-free



Priority: $f_1 \rhd f_2 \rhd f_3$

Weak Envy-freeness



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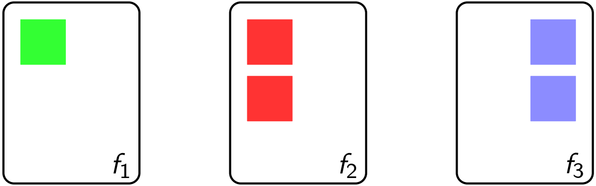
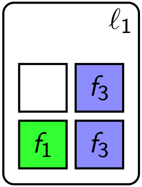
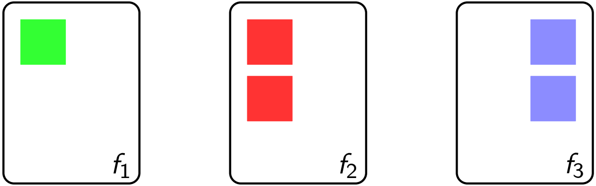


Not envy-free

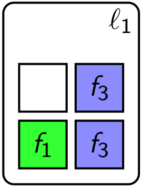


Priority: $f_1 \rhd f_2 \rhd f_3$

Weak Envy-freeness

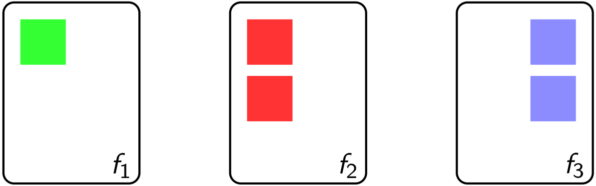


Not envy-free but weakly envy-free

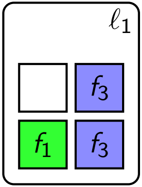


Priority: $f_1 \triangleright f_2 \triangleright f_3$

Weak Envy-freeness



f_2 envies f_3 but f_3 fits even when f_2 is there



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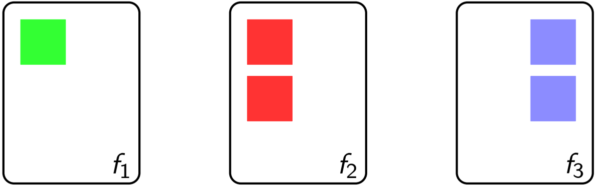

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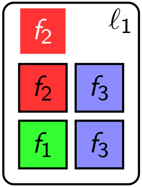

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Cascading Multidimensional DA (CMDA)

Round $k \geq 1$

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- Every family proposes to the locality it prefers among those that have not rejected it

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Theorem

There exists a unique family-optimal weakly envy-free matching and the CMDA algorithm finds it.

Strategy Proofness

The CMDA mechanism is not strategy-proof

- Due to the different sizes, a family may gain by not proposing to localities that reject it

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Solution: make the acceptance rule of localities harsher

- Threshold Multidimensional Deferred Acceptance (TMDA)
- Weakly envy-free and strategy-proof but not family-optimal

Conclusion

Refugee resettlement is a matching problem

- Optimally matching families and localities has long-term consequences
- Multidimensional constraints make it a complex matching problem
- Matching over observables has been done in practice

Solutions to account for preferences

- Using preferences has the potential to further improve the outcome
- Improvement over an endowment

Solutions to account for preferences and priorities

- Localities goodwill is important
- Solution concept: weak envy-freeness

Applications are just starting

- Applications will in turn inform theory